AN ANALYSIS OF THE DEVELOPMENT OF CELESTIAL NAVIGATION

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By

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CHAPTER I

INTRODUCTION

Navigation is the art and the science of conducting a vessel on the water, under the water, or in the air, from one position to another. The problems of navigation are those of direction, distance, and location. Celestial navigation, or observational navigation, is the determination of location by the use of celestial objects.[1] Having measured the altitude of a known celestial body at a known time, an observer determines the relations between latitude, declination, altitude, meridian angle, and azimuth by solving the astronomical or navigational triangle.

In modern times, successful navigation by observation requires six essentials, namely;

1) a map or chart to determine directions and distances,

2) a compass to determine that the vessel is being steered in the desired direction,

3) a sextant, or other instrument, to determine the altitude of celestial objects,

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[1]Navigation may be divided into three main categories. In addition to observational navigation defined above, there is (a) geo-navigation, or navigation by means of landmarks, soundings, and other aids, and (b) electronic navigation, or navigation by means of the various electronic devices such as radar, loran, et cetera.

4) a chronometer to determine the time at which the observation is made,

5) an almanac to determine the celestial coordinates of the observed body, and

6) a method to determine the relationships between the celestial and terrestrial coordinates by solving the navigational or astronomical triangle.

The object of this study is to trace the history and development of these six essentials together with important auxiliary aids, through the ages until after World War II, excluding those that pertain to the fields of geo-navigation and electronic navigation.

This study is confined to celestial navigation for two reasons. In the first place, there is little mathematics involved in geo-navigation, except for the maps. In the second place, electronic navigation is still in an unsettled state. There are several different systems in various stages of development and any analysis would be untimely.

A chronological study of the original sources is to be made as far as possible. When these are not available, commentaries, translations, and later editions are to be used. The division into chapters is to be made on a convenient basis.

CHAPTER II

FROM EARLY TIMES TO 1400 A.D.

The Mediterranean Sea, the great sea that separates Europe from Africa, is approximately 2500 miles in length between the Straits of Gibraltar and the coast of Syria, and approximately 1200 miles in width between Venice and the Gulf of Sidra off the coast of Tripoli. On this sea, navigation was first practiced by the western world.

The credit of being the first to explore the Mediterranean belongs to the Minoans or Cretans.[1] The Egyptians were familiar with the southeastern corner very early in their civilization for the first recorded voyages by them took place at the end of the Third Dynasty (3100 B.C.).[2] The Egyptians did not maintain an interest in maritime enterprises, nor did they establish any colonies; and, by the end of the second millenium B.C., they had renounced seafaring.[3] Tyre, the capital of Phoenicia, claimed to have been the first "to invent navigation and to have taught mankind the art of braving the winds and

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[1]Walter W. Hyde, Ancient Greek Mariners (New York: Oxford University Press, 1947), p. 37.

[2]Ibid., p. 31.

[3]Ibid., p. 32

waves by the assistance of a frail bark."[1]

As early as the fourteenth or fifteenth century B.C., possibly earlier, the Phoenicians were engaged in trade with Egypt, with the island of Cyprus, and with the Libyans on the coast of Africa.[2] Unlike the Egyptians, the Phoenicians established colonies from Crete to Spain. Although most of them were commercial depots, some became important cities, such as Carthage, Utica, and Palermo.[3]

The Greeks[4] and Romans[5] also made voyages on the Mediterranean. They, as well as the Phoenicians, made some use of the sun and stars to determine their course when they ventured beyond the sight of land.[6]

Over the China Sea and the Indian Ocean, the steadiness in the

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[1]Oliver Goldsmith, Alexander Reduces Tyre: Later Founds Alexandria. Rossiter Johnson, Editor-in-Chief, "The Great Events by Famous Historians," (New York: The National Alumni, 1905) Volume II, p. 135.

[2]J. N. Larned, History of the World (New York: World Syndicate Company, Inc., 1915) Volume I, p. 89.

[3]Hyde, op. cit., p. 43.

[4]Ibid., p. 53 ff.

[5]Ibid., p. 134.

[6]R. A. Curtiss, An Account of the Rise of Navigation (Washington: Annual Report of the Smithsonian Institute, 1918) p. 127.

direction of the monsoons was observed. Hippalus, a Greek merchant, discovered about the beginning of the Christian era that he could use the monsoons during the summer months to sail to India and use the counter-monsoons to return during the winter months.[1]

These ancient navigators had manuals, called peripli, which gave the distances along the coast line beginning at a designated place and making a complete circuit. The oldest and most complete periplus which has survived, The Periplus of the Mediterranean and Black Sea, was written sometime between the sixth and fourth century B.C.[2] It is attributed to a Scylax of Caryanda and is sometimes known as the Periplus of Scylax. The Periplus of the Erythraean Sea, a guide for sailing from the Red Sea to India was written about 60 A.D.[3] One of a later date was the Stadiamus or Circumnavigation of the Great Sea compiled sometime between the second and fifth centuries A. D.[4]

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[1]J. Q. Stewart, and N. L. Pierce, Marine and Air Navigation (Boston: Ginn and Company, 1944) p. 391.

[2]Lawrence C. Wroth, The Way of a Ship (Portland, Maine: The Anthoensen Press, 1937) p. 11.

[3]The Periplus of the Erythraean Sea, Wilfred H. Schoff, trans., (New York: Longmans Green and Company, 1912) p. 15.

[4]Wroth, op. cit., p. 12.

Lloyd A. Brown is of the opinion that these peripli were originally designed to accompany a sea chart of some type.[1] Unfortunately, none of these charts are known to exist today.

The historian Herodotus (484 - 425 B.C.) reported that some Phoenicians circumnavigated Africa about 600 B.C. for the Egyptian king Necho. They made the voyage in about three years including stops along the way to plant and harvest crops. The Phoenicians said that when they had rounded Africa and set a northwesterly course the sun was on the right hand. Here was factual evidence that the Phoenicians were south of the equator. Herodotus believed that the voyage was made but doubted that the sun could have been in the position described.[2] In any event, no charts, peripli, or other evidence have been found to substantiate this feat.

There are records of maps in Babylon as far back as 3800 B.C., while cadastral surveys, that is surveys relating to boundaries and subdivisions of land, are recorded on clay tablets made in 2300 B.C.[3]

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[1]Lloyd A. Brown, The Story of Maps (Boston: Little, Brown and Company, 1950) p. 120.

[2]Henry Cary (translator), Herodotus (London: George Bell and Sons, 1891) Book 4, 42, p. 250.

[3]"Maps," Encyclopedia Britannica, Volume XVII, 11th edition, p. 634.

In Egypt during the reign of Rameses II (c. 1250 B.C.) maps of the boundaries of the landed estates were made.[1] The first known map of the world was made by Anaximander (611 - 546 B.C.). He represented the earth as a cylinder suspended from the vault of the heavens;[2] yet, at about this same time, Pythagoras (580 - 497 B.C.), following in the footsteps of Thales (640 - 550 B.C.), may have taught that the earth is spherical.[3] Hicataeus (c. 517 B.C.), who wrote the first geography, The Periodos, sketched the earth as a disc with only two continents, Europe and Asia, of equal size, surrounded by the ocean.[4]

Although the Chinese used a parallel of latitude as early as 1000 B.C., Dicaearcus (326 - 296 B.C.), about seven hundred years later, was the first to draw a parallel across a map, dividing the world into a northern and southern half.[5]

Eratosthenes (276 - 196 B.C.) of Cyrene was appointed

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[1]Brown, op. cit., p. 33.

[2]"Maps," op. cit., p. 634.

[3]David Eugene Smith, History of Mathematics (Boston: Ginn and Company, 1923) Volume I, p. 75.

[4]Brown, op. cit., p. 47.

[5]"Maps," op. cit., p. 634.

Librarian of the library at Alexandria by Ptolemy III (Euergetes I) in 247 B.C. He was the first to measure the size of the earth by a scientific method. He was informed that at the summer solstice at Syene in upper Egypt there was no shadow in the bottom of a well at noon. He knew that on this same day the gnomon on a sundial at Alexandria indicated, from the length of its shadow, that the sun was 1/50 of the circumference of a circle, or 7° 12', from the zenith. Assuming that Syene and Alexandria were on the same meridian, he reasoned that this must be the difference in latitude between the two cities, and hence the circumference of the earth must be fifty times this distance. It is not known how this distance was measured, but Eratosthenes gave its value as 5000 stadia. From this he calculated the circumference of the earth to be 250,000 stadia at the equator, and the length of a degree along a meridian to be 700 stadia. If a stadium was 517 feet[1], then the error in the circumference of the earth was less than five hundred miles, and the error in the length of a degree was less than a mile.[2]

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[1] J. L. E. Dreyer, History of the Planetary Systems from Thales to Kepler. (Cambridge: University Press, 1906) p. 174.

[2] Various authors estimated the length of a stadia differently. It is known that different lengths of the stadia were used at the same time. This accounts for the seeming discrepancies in the values that follow.

Since the latitude of Syene is about 24° 05' and that of Alexandria is about 31° 12', the difference of 7° 7' is a close approximation to Eratosthenes value of 7° 12'.

At this time the obliquity of the ecliptic was about 23° 43'. Hence Syene was not on the Tropic of Cancer. The longitude of Syene is about 3° east of Alexandria.

It would appear, therefore, that a combination of inaccuracies yielded Eratosthenes a very good approximate result.

Eratosthenes was also the author of a treatise on geography. Only parts of it have survived, one being his map. It was in the form of a parallelogram 75,800 stadia east and west, by 46,000 stadia north and south. He drew seven parallels across it and seven meridians perpendicular to them. The seven parallels were drawn through Meroe (near the equator), Syene, Alexandria, Rhodes, Lysimachia (on the Hellespont), the mouth of the Borysthenes (i.e. the Dneipper) and lastly Thule, supposedly the most northern point on the earth. The seven meridians were drawn through the Pillars of Hercules (i.e. Gibraltar), Carthage, Alexandria, Thapsacus (on the Euphrates), the Caspian Gates, the mouth of the Indus, and the mouth of the Ganges. The positions of all of these places were supposed to have been accurately determined by competent authorities.[1]

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[1] "Maps," op. cit., p. 635.

Hipparchus (c. 150 B.C.) objected to this arbitrary network and suggested that the parallels and meridians should be drawn at regular intervals, and that maps should be based upon a regular projection. Hipparchus drew several star maps making use of his suggested projections, but he is not known to have drawn a land map.[1] He developed the spherical trigonometry that he needed in his work on astronomy and worked out a table of chords. He also used a stereographic projection of the celestial sphere on the plane of the earth and compiled a catalogue of 850 fixed stars.[2] Hipparchus was the first to suggest that longitude could be determined by observations of eclipses.[3]

Crates of Mallus (c. 150 B.C.) produced a globe on which he divided the earth into four quarters, each of which was inhabited, thereby unconsciously anticipating the discovery of North and South America and Australia.[4]

Posidonius (130 - 50 B.C.) who was head of the Stoic school at Rhodes, using the arc from Rhodes to Alexandria as a base, determined by observations on the star Canopus the circumference of the

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[1] "Maps," op. cit., p. 635.

[2] Smith, op. cit., I, 119; II, 605.

[3] "Maps," op. cit., p. 635.

[4] Ibid., p. 635.

earth to be 240,000 stadia, thereby establishing the length of a degree as 666 2/3 stadia.[1] It is possible that Posidonius, realizing the inaccuracy of his instruments, may have been responsible for reducing the value of the length of a degree to 500 stadia. This value persisted for centuries, and may have influenced Columbus, for it made the circumference of the earth about 17,000 miles, thus placing India much nearer to Europe than it actually is.[2] Posidonius also noticed the difference between spring and neap tides and suggested that the sun as well as the moon was responsible.[3]

The first general treatise on geography was written by Strabo (c. 40 B.C.). It is not known when he produced his work, but it is known that it was revised sometime between the years 17 and 23 A.D. It was divided into four main divisions; mathematical, physical, political, and historical geography.[4]

Marinus of Tyre (c. 120 A.D.) used Strabo's geography to prepare his charts on which places were located according to their

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[1]Smith, op. cit., II, 371.

[2]"Maps," op. cit., p. 636.

[3]George Sarton, Introduction to the History of Science, Baltimore: The Williams and Wilkins Company, 1927), I, 204.

[4]"Strabo," Encyclopedia Britannica, Volume XXV, 11th edition, p. 974.

latitudes and longitudes. Parallels and meridians were straight lines perpendicular to each other and equally spaced.[1] This map is extinct, but Claudius Ptolemaeus (c. 150 A. D.), usually referred to as Ptolemy, mentions it. From Ptolemy it is learned that Marinus used the value of 500 stadia for the length of a degree. This, in addition to his exaggeration of distances, produced large errors in longitude. His prime meridian was through the "Fortunatae Insulae", that is, the Azores.[2] For many centuries this vicinity was chosen for the prime meridian for it was believed that no land lay to the west of those islands.[3]

Ptolemy produced the Cosmographia in which he drew a world map as well as several sectional maps. He used three projections, one in which the parallels were curved and the meridians straight, one in which both parallels and meridians were curved, and one in which both were straight. His greatest work, The Almagest, contained a summary of the work of Eratosthenes, Posidonius, and others, concerning the size of the earth, the location of certain places, and the

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[1] Ibid, p. 974.

[2] Smith, op. cit., I, 130.

[3] Brown, op. cit., p. 282.

size of islands and countries. He extended Hipparchus' table of chords[1] and increased the latter's star tables to include 1022 fixed stars. He accepted the value of 500 stadia as the length of a degree. It was in the Almagest that he propounded his famous geo-centric theory known as the Ptolmaic System, a theory that was to influence science until the time of Copernicus in the sixteenth century.[2]

In the Ptolmaic System all the apparent motions of the planets, the sun, and the moon, so far as was then observed, could be accounted for by supposing that a stationary earth was at the center of the system and that each planet moved around the circumference of a circle (the epicycle) while the center of this circle (the fictitious planet) moved around the earth on the circumference of a larger circle (the deferent). Figures 1 A and 1 B, on pages 14 and 15, represent the Ptolmaic System. They are not drawn to scale, the deferents being placed at equal distances apart. The epicycle radii of Mars, Jupiter, and Saturn are always parallel to the line joining the earth and the sun. For both Venus and Mercury, Ptolemy assumed that the fictitious planet revolved in its deferent completing one revolution

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[1]Ptolemy's table of chords is a table giving the lengths of the chords subtended by central angles in a circle of radius 60.

[2]Smith, op. cit., I, 130.

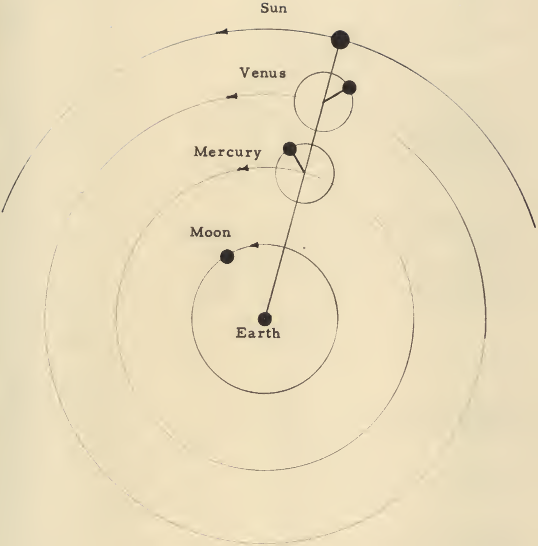

Figure 1 A

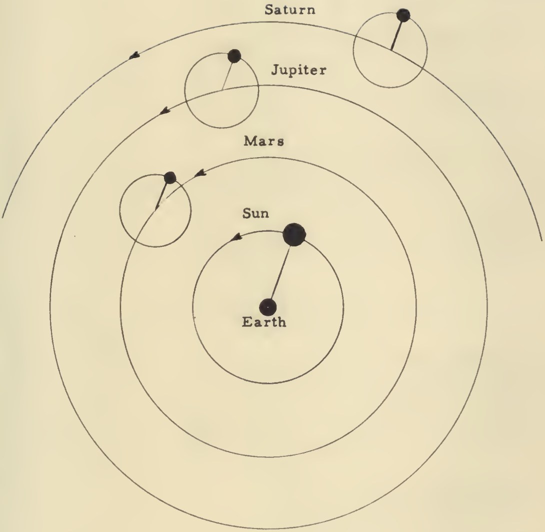

Figure 1 B

annually and always in line with the earth and the sun.[1] The following shows the periods of revolution in the deferent and in the epicycle of the moon and the planets according to Ptolemy.

Celestial Body	Period of Revolution in the Deferent	Period of Revolution in the Epicycle
Moon	27 1/3 days	27 1/3 days
Mercury	1 year	116 days
Venus	1 year	584 days
Sun	1 year	
Mars	1.88 years	780 days
Jupiter	11.8 years	390 days
Saturn	29.5 years	378 days

In order to account for some of the irregularities of the planets' motions, Ptolemy assumed that the deferent and the epicycle were eccentric.

After Ptolemy, a stationary condition followed. During this time science was pursued only by the Arabs who were imitators rather than original investigators. In the ninth century Albatani (850 - 925), the greatest Arabian astronomer, obtained a more accurate measurement of the arc of a meridian than had previously been made. He

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[1] Russell, Dugan, and Stewart, op. cit., I, 243.

corrected the values of precession and the obliquity of the ecliptic, compiled a catalogue of stars, and first used sines and half-chords.[1]

Pope Sylvester II (999 - 1063) accepted as a fact that the earth was a sphere and subscribed to the geo-centric system of the universe, but the map makers, who for the most part were monks, still used the older ideas to form their maps. There were three types, the rectangular, the circular, and the oval. In each the inhabited world was surrounded by the ocean.[2]

About 1000 Abul-Wefa used secants, tangents, and cotangents, and in 1080 Geber introduced the use of the cosine and made some improvements in spherical trigonometry.[3]

Portolan charts, which were the creations of seamen, navigators, and others, were the first modern scientific maps. They were based upon careful and scientific observations. Most of them pertain to the Mediterranean and the Atlantic coast to the north and south of Gibraltar. It is not known when they were first used. The date 1000 A.D. has been suggested, but no portolan chart of that date is known. In fact, none are known to have been drawn prior to 1300 A.D. The

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[1]Charles J. White, The Elements of Theoretical and Descriptive Astronomy (New York: John Wiley and Sons, Inc., seventh edition, revised, 1901) p. 251.

[2]"Maps," op. cit., p. 637.

[3]White, op. cit., p. 251.

oldest bears the date 1311 and the signature of Pietro Visconte. It is reported that the ships of Louis IX used sea charts in 1270.[1]

The Arabs, the Chinese, the Italians, and the Vikings,[2] as well as the Greeks, the Finns, and others,[3] have claimed the honor of inventing the mariner's compass, but none have much evidence to support their claim. Alexander Neckam (c. 1350) in his two treatises De Utensilibus and De Rerum, written in the twelfth century, mentions a needle carried on board ship which if placed on a pivot so that it is free to turn shows the mariners their course when the pole star is hidden.[4]

In 1269 Petrus Peregrinus de Maricourt described a practical compass that could be used at sea; and in 1380 Da Buti described one that had a revolving card with the needle attached to its underside. This is essentially the form of the compass today.[5]

By the year 1400 A.D. the mariner had at his disposal an inaccurate map of the world, a map of the Mediterranean Sea which

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[1]Edward L. Stevenson, Portolan Charts (New York: The Hispanic Society of America, 1911) p. 2.

[2]Wroth, op. cit., p. 23.

[3]"Compass," Encyclopedia Britannica, Volume VI, 11th edition, p. 808.

[4]"Compass," op. cit., p. 808.

[5]Wroth, op. cit., p. 24.

contained gross errors in longitude, but he also had excellent sailing directions, and a fairly good compass for steering whenever landmarks were not available.

CHAPTER III

FROM 1400 TO 1600

Navigation as a science began to be studied by the Portuguese about 1400 A.D. Prince Henry, often referred to as "The Navigator," not only established a school of navigation and astronomy at Sagres near Cape Vincent, the southwestern corner of Europe, but he also built an observatory in order that more accurate tables of the declination of the sun could be obtained.[1] His expeditions discovered the Azores in 1419, rediscovered Cape Verde Islands in 1447, and Sierre Leone in 1460.[2]

The first mention of a Portuguese observation for latitude was in 1462 when Diego de Centra used a quadrant to observe the pole star for this purpose, and about this same time he began to use Ursa Minor to mark the hours of the night.[3] Nineteen years later, Diogo d'Azambrija used the astrolabe for marine purposes.[4]

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[1]"Navigation," Encyclopedia Britannica, Volume XVII, 11th edition, p. 284.

[2]Ibid., p. 284.

[3]Sir Clements R. Markham, "The History of the Gradual Development of the Groundwork of Geographical Science," The Geographical Journal, XLVI (1915) p. 174.

[4]Ibid., p. 176.

Previously, in 1436, a traverse table based upon the trigonometric functions had appeared[1] and sometime between 1450 and 1461 the first almanac was printed.[2]

In 1472 the *Sphaera Mundi* of Johannes de Sacrobosco was published in type. This was a digest of Ptolemy's *Almagest* abridged and translated from the Arabic into Latin about 1230.[3] Two years later, in 1474, Johann Mueller (1436 - 1476) of Konigsberg, Bavaria, known as Regiomontanus, published his *Tabula Directionum* using the value of 23° 30' for the obliquity of the ecliptic.[4]

Portuguese sailors determined their latitude by an observation on the pole star. As their voyages extended south toward and beyond the equator, it became necessary to obtain another method. For this purpose a Mathematical Junta or Committee was appointed in 1481 by King John II who ascended the throne of Portugal that same year. About this time a Jewish scholar, Abraham Zacuto, who had been a professor of astronomy at Salamanca, arrived in Portugal. He

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[1] Wroth, *op. cit.*, p. 32.

[2] "Navigation," *op. cit.*, p. 289.

[3] George F. Chambers, *A Handbook of Descriptive Astronomy* (Oxford, The Clarendon Press, 1877, third edition) p. 766.

[4] Edgar Prestage, *The Portuguese Pioneers* (London: A. and C. Black, Ltd., 1933) p. 318.

brought with him a work in Hebrew entitled Almanac Perpetuum. This work contained tables of the sun's declination, the maximum value being 23° 33'. Joseph Vizinho, a member of the Committee, was a friend of Zacuto and, having obtained his almanac, translated it into Latin.[1] It was used in manuscript form until 1496 when it appeared in print.[2] This Committee suggested the method of measuring the altitude of the sun to determine the latitude when near or below the equator, and also suggested that a simplified form of the astrolabe be designed for this purpose. Such a form of metal or wood, or a combination of metal and wood, must have been produced shortly thereafter, for it is known that these simplified astrolabes were used to determine latitude some years before Columbus sailed on his first voyage to America.[3]

Thus, in 1492, Columbus had available for navigation purposes; a) the Mariner's compass, b) a pair of dividers, c) a quadrant, d) a lead line, e) a sea chart, f) a ruler, g) a traverse table, h) an ordinary multiplication table, and i) an astrolabe which he was not able to use.[4]

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[1]Markham, op. cit., p. 30.

[2]Ibid., p. 25.

[3]Wroth, op. cit., p. 25.

[4]Samuel Morison, Admiral of the Ocean Sea (Boston: Little, Brown and Company, 1944), p. 194.

The earliest navigation manual, Regimento do Estrolabio y do Quadrante was printed in Portugal in 1509. It combined in one manual the Sphaera Mundi of Sacrobosco, a table of the sun's declination for one year based upon Zacuto's almanac, directions for using the tables to determine the latitude by an observation on the sun at meridian height, a list of places with their latitudes, directions for determining latitude by the pole star, and a traverse table to be used to determine the longitude by finding the dead reckoning position.[1]

John Werner (1486 - 1528) in his Ptolemy of 1514 describes the construction and use of the cross-staff and refers to it as an ancient instrument just beginning to be used for navigational purposes. It was in this work that he recommended measuring the angular distance between the moon and certain fixed stars to determine longitude.[2]

R. Gemma Frisius (1508 - 1555) in his work De Principiis Astronomiae, published in 1530, proposed the use of the "Lyttle clocke" to determine the difference in longitude between two places by transporting the instrument from one place to another and noting the difference between the local times of the two places.[3] He also

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[1]Wroth, op. cit., p. 49.

[2]Ibid., p. 26.

[3]Wroth, op. cit., p. 79.

proposed that the distance between meridians be measured on the equator allowing 15° of arc for each hour of time, and that the latitude (i.e. declination) of celestial bodies be measured from the equator not from the ecliptic.[1] In 1569 Tycho Brahe (1546 - 1601) followed the last suggestion.[2]

It is not known when the variation of the compass needle was first recognized. Columbus, in his Journal under date of September 13, 1492, reported a westerly variation.[3] This would seem to imply that he was familiar with an easterly variation. Francisco Falerios (c. 1535) in his work Tratado del Esphera del Arte del Marear published in Seville in 1353, proposed that longitude could be determined by the variation of the compass. To accomplish this he gave complete details based on the calculations of his brother, Ruy Falerio. This idea, which later proved to be false, persisted for over a century.[4]

Two years later, in 1537, Pedro Nunes (1502 - 1578) published Tratado da Esphera coma Theorica do Sol e da Lua, probably the greatest of the early books on navigation. In the third part of this

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[1] "Navigation," op. cit., p. 285.

[2] "Astronomy," Encyclopedia Britannica, 11th edition, Volume II, p. 811.

[3] Morrison, op. cit., p. 203.

[4] Wroth, op. cit., p. 51.

work he called attention to the errors in plane charts, described great circle sailing, adopted Regiomonatanus' value of 23° 30' for the obliquity of the ecliptic, and ignored Falerio's method of determining longitude by the variation of the compass.[1] He realized that the loxodrome[2] is a spiral thereby paving the way for the cylindrical projection of Mercator.[3]

Nunes also described his method of the division of the quadrant by concentric circles - the nonius - a precursor of the vernier. The arc of a large quadrant was furnished with forty-five concentric segments or scales. The outer one was graduated into ninety divisions, the next to eighty-nine, followed by those graduated to eighty-eight, eighty-seven, etc., divisions. The edge of a fine pointer attached to the sights passed among these various divisions. By noting which one the pointer touched, the observer could determine the angle. For example, if it touched the fifteenth division on the sixth scale the angle would be 15/85 of 90°, or 15° 52' 57". Tycho Brahe tried a nonius but found it too cumbersome to use.[4]

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[1]Ibid., p. 53.

[2]A loxodrome is a curve on a surface that cuts meridians at a constant angle.

[3]Prestage, op. cit., p. 325.

[4]A. Wolf, A History of Science Technology, and Philosophy in the XVI and XVII Centuries, (London: George Allen and Unwin, Ltd. second edition, 1950) p. 129.

The famous work of Copernicus (1472 - 1543), De Revolutionibus Orbium Coelestium, in which he proposed his heliocentric theory, as opposed to the geo-centric theory of Ptolemy, was published in 1543.[1] In his theory he showed the apparent motions of the planets could be accounted for just as accurately as in the Ptolmaic System if the sun was placed at the center, and the planets, including the earth, revolved about it. Like Ptolemy, he assumed that the orbits were circular, and found it necessary to have a few small epicycles.[2]

In 1551 Erasmus Reinhold published Tabulae Prutenticae, the first tables based upon the Copernician theory. Meanwhile, Dr. Pedro de Medina's Arte de Navegar, written in Spanish and published in 1545, probably the first textbook on navigation, followed Ptolemy for his astronomy and used Zacuto's value of 23° 33' for the obliquity of the ecliptic.[3] This was followed in 1551 by Breve Compendio de la Sphera de la Arte de Navegar written by Martin Cortes. He also followed Ptolemy and Zacuto.[4]

In 1569 Gerhard Kramer (1512 - 1594), later to be known as

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[1]Smith, op. cit., I, 347.

[2]Giorgio Abetti, The History of Astronomy, Betty B. Abetti, translator, (New York: Henry Schuman, 1952) p. 83.

[3]"Navigation," op. cit., p. 286.

[4]Ibid., p. 286.

Mercator, published his map of the world, the first Mercator projection, which he constructed graphically.[1] Twenty-six years later when Captain John Davis published his famous book, The Seamans Secrets the value of this projection was not yet realized for Davis writes

> . . . yet it cannot bee denyed but Charts for short courses are to uery good purpose for the Pilots vse, and in long courses be the distance neuer so farre, if the Pilot returne by the same course whereby in the first he prosecuted his voyage, his Chart wil be without errour, as an instrument of very great commoditie; but if he returne by any other way then by that which he went forth, the imperfections of the Chart will then appeare to be very great, especially if the voyage be long, or that the same be in the North partes of the world, the farther towards the North, the more imperfect; therefore there is no instrument answerable to the Globe or paradoxall Chart, for all courses and climats whatsoever, by whom all desired truth is most plentifully manifested . . .[2]

Four years later, in 1599, Edward Wright, a professor of mathematics at Caius College, Cambridge, developed the mathematical theory of the projection, which he published in his book Certaine Errors in Navigation Detected and Corrected.[3] In explaining the construction Wright proceeded somewhat as follows: the secant of

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[1] "Maps," op. cit., p. 646.

[2] A. H. Markham, Editor, The Voyages and Works of John Davis, the Navigator, (London: The Hakluyt Society, 1880, series number 59) p. 271.

[3] "Navigation," op. cit., p. 288.

one minute is 10,000,000 which is also the section of one minute of the meridian from the equator. The section of the second minute of the meridian from the equator is the sum of the secant of one minute and the secant of two minutes. Since the latter value is 10,000,002, this sum would be 20,000,002. Then to this add the secant of three minutes, which is 10,000,004, obtaining for the sum the value 30,000,006, thus obtaining the section of the third minute from the equator, and continue this process to obtain additional values of the sections. Wright also realized that this was only an approximation.[1]

In those days the trigonometric functions were lines in a circle of given radius, usually 10,000,000 units. The sine of 90° was equal to the radius and was called the whole sine or the sinus totus.

To aid in finding the latitude at night by an observation on the pole star, the navigator had an instrument known as the Nocturnal, first described by Michael Coignet of Antwerp in 1581. It consisted of two concentric circular plates, the outer about three inches in diameter, and divided into twelve equal parts corresponding to the twelve months. Each month was divided into groups of five days. The inner circle was graduated into twenty-four equal parts, corresponding to the hours of the day, and each of these subdivided into four parts. A handle was attached to the outer circle in such a way that the middle

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[1]H. S. Carslaw, "The Story of Mercator's Map." The Mathematical Gazette, XII (January, 1924), p. 1.

of it corresponded with the day of the month on which the "guards" had the same right ascension as the sun. This was accompanied by tables known as the Regiment of the Pole Star which showed for eight positions of the "guards" the correction to be applied to the altitude of the pole star to obtain the latitude of the observer.[1] Two nocturnals were used at sea; one adapted to the pole star and the first of the "guards" of the Little Bear, that is, Kochab, the other to the pole star and the pointers of the Great Bear, that is Duhbe and Merak.[2]

It was also in The Seamans Secrets that Davis described his backstaff which superseded the cross-staff. To use the astrolabe required three men, one to hold it vertically, one to sight the celestial object whose altitude was desired, and a third to read the angle of elevation. It could not be used to measure other than vertical angles. The cross-staff required the observer to look in two directions at the same time - at the celestial object being observed and at the horizon. As long as the observer was in high latitudes fairly accurate results were obtained, but for observations near or in the tropics, the observation of a blazing sun high in the heavens simultaneously with the horizon was indeed a difficult feat. The Davis backstaff overcame

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[1] "Navigation," op. cit., p. 285.

[2] "Navigation," Encyclopedia Britannica, edition of 1773, (London: Printed for Edward and Charles Dilly, in the Poultry.) MDCCLXXIII, III, 402.

this difficulty, for the observer had the sun at his back so that a shadow fell upon a slit through which the horizon was being observed.[1]

During the two centuries that elapsed from 1400 to 1600, quadrants, astrolabes, cross-staffs, and backstaffs were devised to measure altitudes; tables based upon a more accurate value of the obliquity of the ecliptic were published; several works to aid the navigator were produced in Spain and Portugal; and methods of determining latitude and longitude had been suggested. But the two most outstanding contributions during this time were the mathematical theory of the Mercator projection by Wright, and the heliocentric theory by Copernicus, neither of which were immediately accepted.

Because of gross errors in the terrestrial coordinates of seaports, especially in the values of longitude, those who attempted to use Mercator charts found upon landfall they were far from their destination. The projection was discarded and the older methods used until more accurate values of latitude and longitude were available.

Ptolemy's System remained popular for it put man at the center of the Universe which he considered his rightful place. Furthermore, Copernicus could offer no proof for his theory.

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[1]A. H. Markham, op. cit., p. 330.

CHAPTER IV

FROM 1600 TO 1800

Early in the seventeenth century several advances in astronomy and mathematics were made that proved to be beneficial to navigation. Johannes Kepler (1571 - 1630) announced his three laws:

I. Elliptical Orbit Law: Each planet moves in an elliptical orbit, with the sun at one of its foci.

II. Description of Areas Law: The line joining each planet to the sun sweeps over equal areas in equal intervals of time.

III. Harmonic Law: The squares of the periods of the planets are to each other as the cubes of the major semi-axes of their respective orbits.

The first two were announced in 1609, the third in 1618.[1]

In 1610 Galileo (1564 - 1642) discovered four of Jupiter's satellites (Io, Europa, Ganymede, and Callisto). After observing their occultations he proposed that longitude could be determined from these phenomena and attempted to construct tables for this purpose.[2]

Edmund Gunter (1581 - 1626) published the first tables of the logarithms of sines and tangents of angles to the base ten, thus making logarithms available for navigation.

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[1]"Navigation," op. cit., p. 289.

[2]Georgio Abetti, "Galileo, the Astronomer," Popular Astronomy, Volume LIX, March, 1951, p. 140.

Kepler, in 1627, published the Rudolphine tables of the planets which had been compiled by Tycho Brahe.[1] This was also the year in which Captain John Smith in his Sea Grammar described a "Bittacle" as a square box nailed together with wooden pins because iron pins would attract the compass.[2] This appears to be the first recognition of the deviation of the compass.

Four years later, in 1631, Pierre Vernier (1580 - 1637) invented his device now known as a vernier. About this time Johann Hevelius (1611 - 1687) invented the slow motion or tangent screw and William Gascoigne (c. 1650) attached a telescope to a quadrant for shore use. In 1635 Henry Gellibrand (c. 1650) discovered the annual variation of the compass. Richard Norwood (1590 - 1675) determined by observations and measurements in 1637 that the length of a degree of latitude was 367,176 English feet, an error of about thirty-six feet in a mile, or about six-tenths of one per cent too large.[3] This is not the same Richard Norwood who is credited with the discovery of the dip of the needle, for this was discovered in 1576, fourteen years before the second Richard Norwood was born.

A practical method for the determination of longitude was still

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[1]"Navigation," op. cit., p. 289.

[2]Wroth, op. cit., p. 31.

[3]"Navigation," op. cit., p. 289.

unsolved even though theoretical methods for its determination had been known for centuries. Hipparchus had suggested the method of eclipses,[1] John Werner the method of lunars,[2] R. Gemma Frisius had proposed the "Lyttle clocke"[3] and Galileo the method of occultations.[4]

As early as 1598 King Philip III of Spain offered a reward for a practical solution of the problem. The governments of Holland, Venice, and France, as well as private individuals in several different countries also offered rewards for the same purpose.[5] The astronomers and mathematicians thought the method of lunars seemed the most promising for a practical solution, while the navigators thought that a suitable chronometer would solve the problem.

About 1659 Christian Huygens (1629 - 1695) designed a marine clock that was regulated by a small pendulum. Several were built and used. In 1665 a Major Holmes reported to the Royal Society on the successful performance of several clocks of this type during a voyage from St. Thomas to the Cape Verde Islands.[6]

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[1] Infra, p. 10.

[2] Infra, p. 23.

[3] Infra, p. 23.

[4] Infra, p. 31.

[5] Wroth, op. cit., p. 77.

[6] Wolf, op. cit., p. 117.

In 1675 King Charles II of England gave land at Greenwich, which at that time was a suburb of London, commissioned Sir Christopher Wren to design an observatory, and on March 6th of that year appointed John Flamsteed the first Astronomer Royal. The observatory was established for the purpose of obtaining improved tables of the positions of celestial bodies, especially the moon, and to make them available at least a year in advance.[1]

In April 1686 the Philosophiae Naturalis Principia Mathematica of Sir Isaac Newton (1642 - 1727) was presented to the Royal Society in manuscript form. It was finally printed in 1687 by Edmund Halley (1656 - 1742) at his own expense. Halley did much to popularize Newton's theories in the scientific world by his research on a comet which bears his name. Halley was also interested in the solution of the longitude problem and thought that it could be solved by the variation of the compass. To that end he urged sailors and travellers to make observations and report the results to him.

During the years 1698 - 1700 Halley sailed on a scientific expedition for the sole purpose of gathering additional data. As a result, in 1701, he published the first variation chart. This chart showed the variation for the Atlantic Ocean only, but the following year he published a second chart showing variation all over the world. In both he

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[1] "Navigation," op. cit., p. 289.

used a Mercator projection. These charts were of no value in determining longitude, but they did provide mariners with data for steering any desired course.[1]

In the year 1714 the English Parliament passed an Act to provide a reward for any person or persons who should discover a means of measuring longitude. A body known as the Board of Longitude was appointed and was given the power to grant sums of money to assist experiments and promising inventors. For a method of determining longitude within 60 geographical miles an award of £ 10,000 was offered; within 40 miles £ 15,000; within 30 miles £ 20,000, all to be tested by a voyage to the West Indies and back.[2]

The delay in the solution of the problem was caused primarily for two reasons; first, there was no precise instrument to measure angles; second, tables of the moon's position to any degree of accuracy were not available.

Several instruments had been invented to measure the altitude of a celestial body but none were satisfactory. Thomas Godfrey (1704 - 1749) of Philadelphia, a glazier by trade, became interested in navigation and attempted to improve the Davis backstaff. In 1730 he

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[1] "Sir Isaac Newton," Encyclopedia Britannica, 11th edition, Volume XIX, p. 587.

[2] Curtiss, op. cit., p. 130.

devised a form of it which he called a mariner's bow. He then attempted to improve a forestaff which led him eventually to consider a double reflecting instrument, and in November of that year he completed a double reflecting quadrant. This instrument was tested at sea and found to be very exact.[1] During the same year, John Hadley (1682 - 1744), who was vice-president of the Royal Society, also invented a double reflecting quadrant, and later a double reflecting sextant, with a spirit level attached.[2]

In 1735 John Harrison (1693 - 1776) completed his first chronometer and applied for the longitude prize. He was sent on a voyage to Lisbon and back to test it. From the encouraging results obtained, the Board granted him L 500 in order that he might continue his experiments. Three years later he completed his second chronometer which was similar to the first with some improvements. This was followed by a third, but it, like the second, was never tested at sea. In 1759 he completed his fourth and finest chronometer. It resembled a watch about five inches in diameter, with an hour hand, a minute hand and a sweep second hand. It was tested in 1761 on a voyage to the West Indies and return. Although this voyage lasted over five months, the

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[1] H. D. McGuire, "The American Inventor of the Reflecting Quadrant," U. S. Naval Institute Proceedings, Volume LXV, 1939, pp. 1171 ff.

[2] Sir H. S. Jones, "The Development of Navigation," Popular Astronomy, Volume LVI, 1948, p. 265.

total error of the chronometer, was less than two minutes.

The Board of Longitude, astounded by the results, refused to pay the full prize, but did advance Harrison L 2500. Harrison agreed to another trial which took place two years later. On a seven weeks trip to Barbados the chronometer gained 38.4 seconds. The Board still refused to pay unless Harrison would give them the secrets of the mechanism, and also the previous three chronometers that he had invented. Harrison at last complied, but he did not receive the full amount of the award until 1773, three years before his death.[1]

Tobias Mayer (1723 - 1762) in Germany had worked out a lunar theory and had calculated tables which were a great improvement on previous ones. He sent them to the Admiralty in England in 1755. They were tested by William Bradley (1693 - 1762) who was the Astronomer Royal at the time, and found to be accurate, in general, within one minute of arc. They were not published, however, until 1770.[2]

Nevil Maskelyne (1732 - 1811) was sent to St. Helene in 1761 on an expedition to observe the transit of Venus. On the voyage there and back he took several observations on the moon. Using Mayer's

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[1]Alfred Gelligras, "John Harrison, a Pioneer in Navigation," Popular Astronomy, Volume LIII, 1945, p. 425.

[2]Sir Harold S. Jones, "The Development of Navigation," Popular Astronomy, Volume LVI, 1948, p. 265.

tables he considered that his observations and calculations would yield a longitude within about a degree and a half. In 1763 on his return to England, he published the British Mariner's Guide in which he explained the method of determining longitude within about a degree by observing the moon and sun, planet, or star by Hadley's quadrant. He suggested the latitude and longitude be computed for every twelve hours, and the distance from the sun, and from a star on each side of the moon be calculated for every six hours and published in advance.[1] This date, 1763, is the one usually associated with the Time Sight, that is, any observation on a celestial body that yields the longitude or the time.

In 1765 Maskelyne became the Astronomer Royal. Shortly thereafter he began organizing the publication of a nautical almanac, and in 1766 the first British Nautical Almanac appeared. Maskelyne believed that the method of lunars offered the best solution to the longitude problem.[2]

The lunar method of determining longitude is as follows: the angular distance from the moon to the sun, planet, or star nearly in the ecliptic, is measured. This apparent distance is then reduced to the corresponding geo-centric distance. The rapid movement of the

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[1] "Navigation," op. cit., p. 292.

[2] Ibid., p. 292.

moon in its orbit is such that the Greenwich Mean Time may be found, for this distance will be the same only at very nearly the same absolute time. The difference between the Greenwich Mean Time and the Local Mean Time yields the observer's longitude. The American Ephemeris and Nautical Almanac tabulated, until the Ephemeris of 1911,[1] published in 1909, the moon's geocentric distances from the sun, Venus, Mars, Jupiter, Saturn, and the nine bright stars, Aldebaran, Altair, Antares, Fomalhaut, Hamal, Markab, Pollux, Regulus, and Spica, at three hour intervals for every day of the year.[2]

A formula for the computation of lunar distances may be developed as follows: in Figure 2, page 40, Z is the observer's zenith, WABE is the horizon, m and s the observed places of the moon and sun respectively (in this discussion the sun will be used, but the results hold for a planet or a star), M and S the true places of the moon and sun respectively. M is nearer the observer's zenith than m for the moon's parallax is greater than its refraction, while for the sun, the refraction is greater than the parallax and consequently s is nearer the zenith than S. The values of Am, Bs, and ms are obtained by observation, from which AM and BS are obtained by correcting for

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[1] The American Ephemeris and Nautical Almanac. (Washington: Government Printing Office, 1912) p. iii.

[2] The American Ephemeris and Nautical Almanac. (Washington: Government Printing Office, 1911) pp. xiii ff.

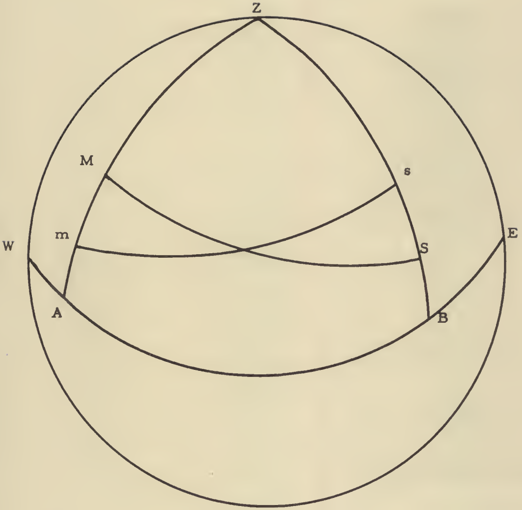

Figure 2

refraction, dip, parallax, and, if necessary, for semi-diameter.

Let $d =$ the apparent distance, ms

$D =$ the true distance, MS

$h =$ the apparent altitude of the moon, Am

$h' =$ the apparent altitude of the sun, Bs

$H =$ the true altitude of the moon, AM

$H' =$ the true altitude of the sun, BS

Using the cosine law on the triangles Zms (Figure 3, page 42) and ZMS (Figure 4, page 42)

$$\cos d = \cos (90 - h) \cos (90 - h') + \sin (90 - h) \sin (90 - h') \cos Z$$

$$\cos D = \cos (90 - H) \cos (90 - H') + \sin (90 - H) \sin (90 - H') \cos Z$$

from which

$$\cos Z = \frac{\cos d - \sin h \sin h'}{\cos h \cos h'}$$

and

$$\cos Z = \frac{\cos D - \sin H \sin H'}{\cos H \cos H'}$$

Solving these two equations simultaneously a formula for the cosine of the true distance may be obtained, thus

$$\cos D = (\cos d - \sin h \sin h') \frac{\cos H \cos H'}{\cos h \cos h'} + \sin H \sin H'.$$

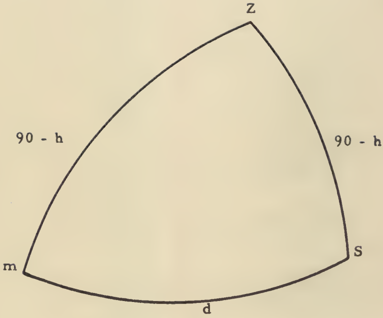

Figure 3

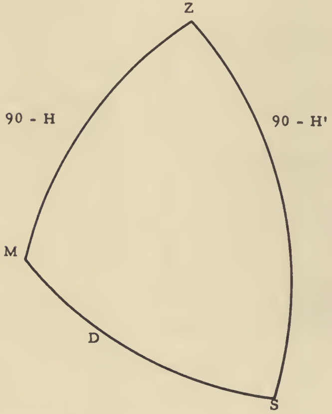

Figure 4

This may be put in the form

$$\cos D = 2 \cos 1/2\,(h + h' + d) \cos 1/2 \left[(h + h') - d\right]$$

$$\frac{\cos H \cos H'}{\cos h \cos h'} - \cos (H + H')$$

This is a formula for the solution of the lunar distance problem. From it, after subtracting each side from unity and using formulas for sin 1/2D and cos 1/2D, and extracting the root, Borda's formula

$$\sin 1/2D = \cos 1/2\,(H + H') \cos \phi$$

is obtained, where

$$\cos^2 \phi = 1 - \frac{\cos 1/2\,(h + h' + d) \cos 1/2 \left[(h + h') - d\right]}{\cos h \cos h'}$$

$$\frac{\mathrm{Cos}\, H \cos H'}{\cos^2 1/2\,(H + H')}$$

Four observers are necessary for an accurate determination of longitude by this method; one to measure the distance between the celestial objects, a second to measure the altitude of the moon, a third to measure the altitude of the sun, and a fourth to note the time of the three simultaneous observations.

If four observers were not available, a single observer might obtain a lunar by using the following sequence: 1) observe the altitude of the sun, 2) observe the altitude of the moon, 3) measure the angular distance between the sun and the moon, 4) observe the altitude of the moon, 5) observe the altitude of the sun. The mean of the altitudes of the sun and the moon will give the approximate altitudes which the sun and moon had when the distance was measured.

The moon moves at the rate of about a degree in arc in two hours of time, or one minute of arc in two minutes of time. Hence an error of one minute of arc in observing the distance would make an error of two minutes in time, or thirty miles of longitude at the equator.

Even with available tables, the method of lunars require lengthy calculations. To assist the navigator, to determine his longitude by Chauvenet's method,[1] fourteen special tables were printed in Bowditch. The table number, title, and computing formula follow.

Table	Title	Formula
I	Mean Reduced Refraction for Lunars using a barometer of 30 inches and a Fahrenheit temperature of 50 degrees	$r' = \frac{r}{\cos h} = \frac{k}{\sin h}$
II	Log A, for computing the First Correction of the Lunar Distance	$A = (K')^2 \frac{\sin(h + 1/2\Delta h)}{\sin h}$
III	Log B, for computing the First Correction of the Lunar Distance	$B = K' \frac{\sin(2H - \Delta H)}{\sin 2H}$
IV	Log C, for computing the First Correction of the Lunar Distance	$C = \frac{\sin(H - \Delta H)}{\sin H}$
V	Log D, for computing the First Correction of the Lunar Distance	$D = \frac{\sin(2h + \Delta h)}{\sin h}$
VI	Second Correction of the Lunar Distance	$\Delta_2 d = -1/2\Delta_1 d^2 \sin 1'' \cot d$

[1] William Chauvenet, A Manual of Spherical and Practical Astronomy, (Philadelphia, J. B. Lippincott and Company, 5th edition, revised and corrected, 1863) p. 402 ff.

Table	Title	Formula
VII A	For Finding the Correction of the Lunar Distance for the Contraction of the Moon's Semi-diameter	$g = \frac{\Delta s_o}{(p - r')^2 \cos^2 h} f$
VII B	Contraction of the Moon's Semi-diameter	$\Delta s = (A' + B')^2 \frac{g}{f}$
VIII A	For Finding the Correction of the Lunar Distance for the Contraction of the Sun's Semi-diameter	$G = \frac{(R' - P)^2 \cos^2 H}{\Delta S_o} F$
VIII B	Contraction of the Sun's Semi-diameter	$\Delta S = (C' + D')^2 \frac{F}{G}$
IX	Logarithms of Small Arcs in Space or Time	The logarithms of the arc in seconds
X	The Correction Required on account of Second Differences of the Moon's Motion	$- \frac{t(180^m - t) \Delta Q}{2.60576}$
XI A	For Finding the Value of N for Correction for the Compression of the Earth for the Moon - i.e. the First Part of N	$a = -N'p' \sin \delta \cot d$
XI B	For Finding the Value of N for Correction for the Compression of the Earth for any other body - i.e. the Second Part of N	$b = N'p' \sin \Delta \operatorname{cosec} d$

where

$A' = A(p - r') \sin h \cot d$

$B' = -B(p - r') \sin H \operatorname{cosec} D$

$C' = -C(R' - P) \sin H \cot d$

$D' = D(R' - P) \sin h \operatorname{cosec} d$

d = the apparent distance of the moon from the sun

δ = the moon's declination

Δ = the sun's declination

$\Delta_1 d = A' + B' + C' + D'$

$\Delta_2 d = -1/2 \Delta_1 d^2 \sin 1'' \cot d$

ΔH = the difference of the sun's apparent and true altitudes

Δh = the difference between the moon's apparent and true altitudes

ΔQ = the increase in 3 hours of time of proportional logarithms of the difference of the distances given in the Ephemeris

ΔS = the contraction of the inclined semi-diameter of the sun

ΔS_o = the contraction of the vertical semi-diameter of the sun

Δs = the contraction of the moon's semi-diameter

Δs_o = the contraction of the moon's vertical semi-diameter

F = 1/200 (an arbitrary factor chosen to give G convenient integral values)

f = 18,000,000 (an arbitrary factor chosen to give g convenient integral values)

H = the sun's apparent altitude

h = the moon's apparent altitude

K' = 1.00029

N' = .006686

P = the sun's reduced parallax

p = the moon's reduced parallax

p' = 57' 30" (the assumed mean value of the moon's horizontal parallax)

R' = the sun's reduced refraction

r' = the moon's reduced refraction

t = the time in minutes

In addition to these fourteen special tables, six other tables are needed. In the 1901 edition of Bowditch they were numbered 14 (correction for dip), 18 (correction for the moon's semi-diameter and augmentation), 19 (correction for the moon's horizontal parallax), 21 (correction for mean refraction used in table I), 22(correction for mean temperature used in table I), and 44 (logarithms for trigonometric functions).

The following data are required:

1. Latitude and approximate longitude of the observer.
2. Approximate local time.
3. Greenwich Mean Time of the observation.
4. Apparent distance of the moon's bright limb from a star or planet, or from the nearer limb of the sun.
5. Apparent altitude of the moon's upper or lower limb above the horizon.
6. Apparent altitude of a star, planet, or lower limb of the sun above the horizon.
7. Barometric pressure in inches of mercury.
8. Temperature in degrees Fahrenheit.
9. Height of eye above sea level.
10. The index correction of the sextant.

The computation consists of three parts.

Part I. Preparation of the Data.

1. Enter the Nautical Almanac with the Greenwich date and take out the moon's semi-diameter and horizontal parallax; if the sun has been observed, take its semi-diameter; if a planet, take only its parallax.

2. Correct the altitude of the moon for dip (table 14), semi-diameter, and augmentation (table 18). Label this the moon's apparent altitude.

3. Correct the altitude of the sun (planet or star) for dip (table 14), and for semi-diameter in the case of the sun. Label this the sun's apparent altitude.

4. Correct the apparent distance for the moon's augmented semi-diameter and the sun's semi-diameter. Label this the apparent distance.

5. Correct the moon's horizontal parallax (table 19). Label this the moon's reduced parallax.

6. Enter table I with the moon's apparent altitude obtaining the correction for mean refraction and temperature and correct this value for barometric pressure (table 21) and for the temperature (table 22). Label this the moon's reduced refraction.

7. Subtract the result of 6 from that of 5. Label this the moon's reduced parallax and refraction.

8. Enter table I with the sun's apparent altitude obtaining the correction of mean refraction and temperature and correct this value for barometric pressure (table 2 1) and for the temperature (table 22). Label this the sun's reduced parallax and refraction.

This completes Part I.

Part II. Computation of the True Distance.

1. Using tables II, III, IV, and V, take out the logarithms of A, B, C, and D, and place each at the head of a column marked accordingly.

2. From table IX find the logarithm of the moon's reduced parallax and refraction and put it in column A and in column B.

3. From table IX find the logarithm of the sun's reduced parallax and refraction and enter it in column C and in column D.

4 Find the log sin moon's apparent altitude (table 44) and put it in column A and in column D.

5. Find the log sin sun's apparent altitude (table 44) and put it in column B and in column C.

6 Find the log cot apparent distance (table 44) and put it in column A and in column C.

7. Find the log cosec apparent distance and put it in column B and in column D.

8. The sum of the quantities in column A is the logarithm of the First Part of the Moon's Correction. Enter table IX with this value and find its anti-logarithm.

9. The sum of the quantities in column B is the logarithm of the Second Part of the Moon's Correction. Enter table IX with this value andfind its anti-logarithm.

10. The sum of the quantities in column C is the First Part of the Sun's Correction. Enter table IX with this value and find its anti-logarithm.

11. The sum of the quantities in column D is the logarithm of the Second Part of the Sun's Correction. Enter table IX with this value and find its anti-logarithm.

12. Combine the results of steps 7 and 8 to obtain the Moon's Whole Correction.

13. Combine the results of steps 9 and 10 to obtain the Sun's Whole Correction.

14. Combine the results obtained in steps 12 and 13 to obtain the First Correction of Distance.

15. Enter table VI with the result obtained in step 13 and the apparent distance as arguments and find the Second Correction of Distance.

16 Enter table VII A with the moon's apparent altitude and the moon's Reduction in Parallax and Refraction and take out the number found.

17. With this value and the moon's Whole Correction enter table VII B and take out the contraction which is to be applied to the apparent distance.

18. Enter table VIII A with the sun's apparent altitude and the sun's Reduced Parallax and Refraction and take out the number found.

19. With this value and the sun's Whole Correction enter table VIII B and take out the contraction which is also to be applied to the apparent distance.

20. From the Nautical Almanac take the declinations of the observed bodies to the nearest degree.

21. Enter table XI A with the declination of the moon and the apparent distance and take out the First Part of N.

22. Enter table XI A with the declination of the sun and the apparent distance and take out the Second Part of N.

23. Combine the results obtained from steps 21 and 22. Enter table IX and find the logarithm of this number. This is log N.

24. Add the logarithm of N and logarithm of the sine of the latitude to obtain the logarithm of the required correction for compression. Find its anti-logarithm from table IX.

The result is the True Distance.

Part III. Computation of Greenwich Mean Time

1. In the Nautical Almanac find the two distances between which the True Distance falls. Take out the first of these and its proportional logarithm and the Greenwich Time.

2. Find the difference between the Almanac Distance and the True Distance and to the logarithm of this difference found in table IX add the proportional logarithm from the Nautical Almanac. This is the logarithm of an interval of time to be added to the hours of Greenwich time to give approximate Greenwich time.

3. To find the True Greenwich Mean Time, take the difference between the two proportional logarithms in the Almanac corresponding to the two distances in step 1, above. With this difference and the interval of time (table IX) found in step 2, above, enter table X and take out the number of seconds. This value applied to the approximate Greenwich Time gives the True Greenwich Mean Time.

The navigator is now ready to find his longitude. It is the difference between True Greenwich Mean Time and Local Mean Time.[1]

The determination of longitude by the method of lunars is a formidable problem. Nevertheless it was used extensively at sea to

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[1]N. Bowditch, The American Practical Navigator, (Washington: U. S. Navy Hydrographic Office, 1906), p. 288.

correct the chronometer, and the special tables and the method were given in Bowditch until the 1912 edition.

At the close of the eighteenth century, the six essentials[1] for successful navigation were available. The mariner had Mercator charts which were excellent as long as he remained within sixty degrees of the equator; a fairly reliable compass; a double reflecting sextant and quadrant; a chronometer; an almanac; and several methods to determine his position. Improvements in each of these six essentials were to be expected, and better knowledge of tides, winds, and currents were needed.

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[1] *Infra*, p. 1.

CHAPTER V

FROM 1800 TO 1900

In 1802 the first edition of The New American Practical Navigator by Nathaniel Bowditch (1773 - 1838) was published. Some years previously Bowditch had found over eight thousand errors in the tables in Moore's Practical Navigator and over two thousand errors in the tables in the second edition of Maskelyne's Requisite Tables, two books that were used extensively at that time. Bowditch decided therefore, not only to publish tables of correct values, but also to collect into one volume all that would be necessary for a complete system of practical navigation.[1] The publication was an immediate success and before his death sold over thirty thousand copies in ten editions.[2]

In the first edition a method of lunars was given that was an improvement over previous approximate methods. All the corrections were additive, thereby eliminating several different cases. Bowditch also invented another method of correcting the distance and improved upon a third. With the computing and publishing of several special tables to be used in determining the longitude by the method of

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[1] Nathaniel Bowditch, The New American Practical Navigator (New York: E & G. W. Blunt, 17th edition, 1847) p. iii.

[2] "Bowditch" (Washington: U. S. Hydrographic Office, revised edition of 1938) p. 3.

lunars, Bowditch made a great contribution to navigation.

After the publication of an approximate method of lunars by William Chauvenet (1820 - 1870) in the appendix of the American Ephemeris and Nautical Almanac for the year 1857, no other method was used officially by the United States Navy as long as lunars was practiced (1912). Chauvenet had published this method about seven years previously.[1]

At the equator a degree of longitude is the same as a degree of latitude, assuming the earth is a sphere. As the poles are approached, the distance between corresponding degrees of latitude remains the same while the distance between corresponding degrees of longitude decreases. On a Mercator chart the distances between meridians of equal longitude are constant. To preserve the relations that exist at different parts of the earth's surface between parallels of latitude and meridians of longitude, it is necessary to expand the latitudes.

In Figure 5, page 55, O is the center of the earth, assumed to be a sphere of radius R. A and B are two places located on the same parallel of radius r. PC and PD are meridians through A and B respectively. The arc CD along the equator is the difference of longitude between the two meridians.

In a Mercator chart the meridians and parallels form a

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[1]Chauvenet, op. cit., p. 402.

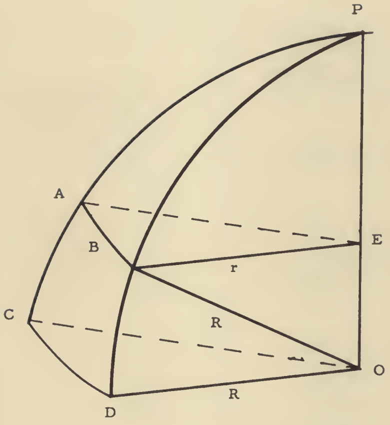

Figure 5

rectangular network. The representation of a spherical surface on a plane in this manner requires that the distance AB be expanded. From the figure it is evident that

$$\frac{CD}{AB} = \frac{OD}{EB} = \frac{R}{r} = \sec BOD = \text{the secant of the latitude}$$

The distance between equal units of longitudes is constant, hence the distance between corresponding units of latitude must be expanded in proportion to the secant of the latitude.

The length of the arc of a meridian expressed in units chosen to represent one minute of arc of the equator is defined as the number of meridional parts,[1] corresponding to that latitude.

Wright obtained his values from a formula which is essentially[2]

$$\text{meridional parts for } L^{\circ} = \sec 0^{\circ} + \sec 1'$$
$$+ \sec 2' + \sec 3' + \dots + \sec (L^{\circ} - 1').$$

In the early editions of Bowditch, as The New American Practical Navigator was soon called, the tables of meridional parts were computed from the formula

$$m = T(.0007915704468)$$

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[1]Benjamin Dutton, Navigation and Nautical Astronomy (Annapolis, Maryland, United States Naval Institute, eighth edition, 1943) p. 8.

[2]Infra, p. 27.

where

$m =$ the number of meridional parts of that latitude in miles

$T =$ log tangent less radius of 1/2 the latitude plus 45^{o} taken to seven digits multiplied by 10^{7}.

In present day notation T is $10^{7} \log \tan (45^{o} + 1/2\ L)$. Values were tabulated for every degree and minute from the equator to $83^{o}\ 59'$ to the nearest unit.

For greater accuracy the tables contained in later editions are based on the Clark Spheroid of 1880. These tables contain the meridional parts for every degree and minute of latitude from the equator to 80^{o} tabulated to one decimal place and computed from the formula[1]

$$m = \frac{a}{M} \quad \log \tan(45^{o} + 1/2L) - a(e^{2} \sin L + 1/3\ e^{4} \sin^{3} L + 1/5\ e^{6} \sin^{5} L + \ldots)$$

where

$m =$ the number of meridional parts of that latitude in miles

$a = 10800/\pi = 3437'.74677$, the equatorial radius

$M = .4342925$, the modulus of common logarithms

$c = 1/293.465$, the compression factor

$e = \sqrt{2c - c^{2}} = 0.0824846$

Since the difference in the two formulas is the second term, the

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[1] "Bowditch," op. cit., part II, p. 5.

values given in the early editions were too large amounting to a difference of four miles at latitude 10^{o} to a difference of twenty miles at latitude 60^{o}.

A navigator had at his disposal several methods of determining his longitude, azimuth, and latitude. The usual practice was to observe the sun in the morning when on or near the prime vertical (Time Sight), next to observe the sun when on the meridian (Meridian altitude) or near the meridian (Reduction to the Meridian), thirdly, to observe the sun again in the afternoon when on or near the prime vertical, and lastly, during evening twilight, to observe Polaris if the position was not too far south.

The observations on or near the prime vertical determine the longitude of the observer and the azimuth of the body, the others determine the latitude.

For the morning Time Sight the navigator used the dead reckoning (D. R.) latitude if he considered it reliable, and computed the longitude and azimuth. Usually, however, he waited until the noon latitude was obtained, then worked his traverse backward to obtain his morning latitude. With this value he then computed his morning longitude and azimuth. From this morning position he worked his traverse forward to obtain his noon position. For the afternoon Time Sight, he carried forward his noon latitude for the afternoon latitude to use to compute the afternoon longitude. The evening observation on

Polaris, if available, gave him a check on his latitude at that time.

The longitude may be computed from the formula

$$\sin^2 1/2t = \sec L \operatorname{cosec} p \cos s \sin(s - h)$$

where

t = the meridian angle

L = the latitude of the observer

p = the polar distance of the sun

$s = 1/2(h + L + p)$

h = the observed altitude

As more and more iron was used in the construction of ships, the necessity of a true bearing to determine the deviation of the compass became increasingly important. The purpose of computing the azimuth is to obtain the deviation by comparing the azimuth observed at the time of the sight with the computed azimuth. At least three methods were available, the Time Azimuth, the Altitude Azimuth, and the Time and Altitude Azimuth.

For the Time Azimuth a formula is[1]

$$Z = X \sim Y$$

where

$\tan X = \sin D \operatorname{cosec} S \cot 1/2t$

$\tan Y = \cos D \sec S \cot 1/2t$

---

[1]The symbol $\sim$, which is read "on to," in the case of $A \sim B$, means either $A + B$, $A - B$, or $B - A$, depending upon the circumstances.

Z = the true azimuth

S = 1/2 sum of the polar distance and co-latitude

D = 1/2 difference of polar distance and co-latitude

t = the hour angle.

For the Altitude Azimuth a formula is

$$\sin^2 1/2 Z = \sin(s - L) \sin(s - h) \sec h \sec L.$$

Another is

$$\cos^2 1/2 Z = \cos s \cos(s - p) \sec h \sec L$$

where

Z = the azimuth

$s = 1/2(h + L + p)$

p = the polar distance

h = the observed altitude

L = the latitude

in both formulas.

For the Time and Altitude Azimuth a formula is

$$\sin Z = \sin t \cos d \sec h,$$ [1]

where Z, t, d, and h have the same meaning as above.

If the body is near the prime vertical the observer may be at a loss to know whether the azimuth is to be measured from the north or south point of the horizon. However, at sea the approximate azimuth is usually known. If in doubt, the altitude on the prime vertical can be

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[1] "Bowditch," op. cit., edition of 1906, pp. 110 ff.

computed by the formula

$$\sin h = \sin d \operatorname{cosec} L$$

where h, d, and L have the usual meaning. Then if the observed altitude is less than the computed altitude, the bearing is on the side of the elevated pole.[1]

Several methods were available to determine latitude.[2] In addition to the Meridian Altitude, Reduction to the Meridian, and Polaris methods, the Phi prime Phi double prime was popular and used extensively.

Since the latitude of the observer is the zenith distance of the equator, an observation of a celestial body on the meridian, after applying the usual corrections and allowing for the declination, will yield the latitude.

The celestial body may not always be available when on the meridian. To overcome this difficulty the method of Reduction to the Meridian was devised. From the fundamental equation

$$1)\quad \sin h = \sin L \sin d + \cos L \cos d \cos t$$

using the substitution

$$\cos t = 1 - \sin^2 1/2t$$

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[1] "Bowditch," op. cit., edition of 1906, p. 112.

[2] William Chauvenet, A Manual of Spherical and Practical Astronomy (Philadelphia: J. B. Lippincott and Company, fifth edition, revised and corrected, 1863) p. 304.

equation 1) becomes

$$\sin h = \sin L \sin d + \cos L \cos d$$
$$- 2 \cos L \cos d \sin^2 1/2t$$

or

$$\sin h = \cos(L \sim d) - 2 \cos L \cos d \sin^2 1/2t.$$

Since $L \sim d$ is equal to $90^\circ - h_o$, where h_o is the meridian altitude of the body at some place in the same latitude as the observer at the same instant when the body's declination is d,

$$\sin h = \sin h_o - 2 \cos L \cos d \sin^2 1/2t$$

and

$$\sin h_o = \sin h + 2 \cos L \cos d \sin^2 1/2t.$$

From the last equation,

$$\sin h_o - \sin h = 2 \cos L \cos d \sin^2 1/2t$$

or

$$\cos 1/2(h_o + h) \sin 1/2(h_o - h) = \cos L \cos d \sin^2 1/2t.$$

Since the difference between h_o and h is small

$$\cos 1/2(h_o + h) = \cos h_o.$$

Also

$$\cos h_o = \sin (L \sim d).$$

The value $(h_o - h) = \underline{a}$ is the Reduction to the Meridian, hence

$$\sin 1/2a = \cos L \cos d \sin^2 1/2t \operatorname{cosec} (L \sim d).$$

It is assumed that the body is near the meridian, therefore $\underline{a}$ and $\underline{t}$ are small. As a result

$\sin 1/2a = 1/2\, a'' \sin 1''$, where <u>a</u> is expressed in seconds of arc, and

$\sin 1/2t = 1/2(15\, t) \sin 1''$, where <u>t</u> is expressed in seconds of time.

Therefore

$$a'' = 112.5\, t^2 \cos L \cos d \operatorname{cosec} (L \sim d) \sin 1''.$$

If the hour angle is expressed in minutes of time, then

$$a'' = 112.5\, (60\, t)^2\, (.000004848 \cos L \cos d \operatorname{cosec} (L \sim d))$$

or

$$a'' = \frac{1''.96349\, t^2 \cos L \cos d}{\sin (L \sim d)}$$

To accomplish the Reduction to the Meridian the formula is

$$H = h + at^2$$

where

H = the meridian altitude at the time of the observation

h = the observed altitude

a = the change in altitude in seconds of arc in one minute of time from the meridian

t = the interval from meridian passage.

Bowditch published two tables (tables 26 and 27 in the older editions, tables 29 and 30 in the later editions) to facilitate the reduction. The first table (26 or 29) gives values of <u>a</u> for one minute of time computed from the formula

$$a = \frac{1''.9635 \cos L \cos d}{\sin (L \sim d)}$$

derived above. The second table (27 or 30) gives values of at^2. To use the tables, enter the first table with the latitude and declination as arguments to obtain a value for a, then enter the second table with this value and the number of minutes from meridional passage to obtain values of at^2. If the noon latitude is desired allowance for the run of the ship must be made.

The altitude of the elevated pole is equal to the latitude of the observer. At the present time (1953) there is no star close to the south celestial pole, but Polaris is less than a degree from the north celestial pole. To determine the latitude from an observation on Polaris requires a correction which depends upon the position of Polaris with respect to the pole. Tables for this correction are given in the various almanacs and are based upon the formula

$$c = p \cos t - 1/2 p^2 \sin^2 t \tan h \sin 1''$$

where

c = the correction to be applied to the observed altitude in seconds of arc

p = the complement of the declination in seconds of arc

t = the hour angle

h = the observed altitude.

In Figure 6, page 65

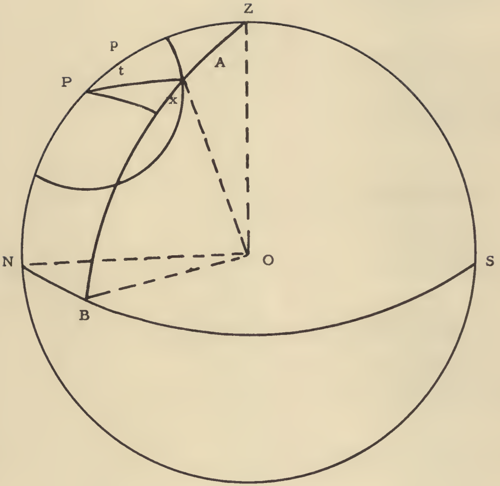

Figure 6

Z = the observer's zenith

P = the north celestial pole

NS = the observer's horizon

O = the center of the earth

p = the complement of the declination of Polaris

h = the observed altitude, angle BOA

A = the position of Polaris at the observed time

L = the latitude, or $h \pm x$, where $x \doteq p$

In the law of cosines,

$\sin h = \sin (h - x) \cos p + \cos (h - x) \sin p \cos t$, if $\sin (h - x)$ and $\cos (h - x)$ are expanded in a Taylor series and sin p and cos p in a Maclaurin series, neglecting powers beyond the fourth, and substituted in the above formula the result is

$$x = p \cos t - 1/2(x^2 - 2\,px \cos t + p^2) \tan h$$
$$+ 1/6(x^3 - 3px^2 \cos t + 3\,p^2 x - p^3 \cos t)$$
$$+ 1/24(x^4 - 4px^3 \cos t + 6\,p^2 x^2$$
$$- 4\,p^3 x \cos t + p^4) \tan h\,.$$

By a series of approximations,

$$x = p \cos t - 1/2\,p^2 \sin^2 t \tan h$$

or in seconds of arc

$$x = p \cos t - 1/2\,p^2 \sin^2 t \tan h \sin 1''.$$

The Phi prime Phi double prime method of determining latitude is limited to conditions where the body is within three hours of meridian

passage, not more than 45° from the meridian in azimuth, and with a declination of at least 3°. If the meridian angle is six hours, or if the declination is zero, the method cannot be used.

The astronomical triangle projected on the plane of the celestial horizon is shown in Figure 7, page 68. R is the length of the perpendicular from the body to the observer's meridian, intersecting it at point A. Phi double prime is the latitude of point A and Phi prime is the distance from the zenith to Point A.

From Napier's Rules of Circular Parts, the following are derived:

$$\sin R = \cos d \sin t$$

$$\sin d = \cos R \sin \phi''$$

$$\sin h = \cos \phi' \cos R$$ [1]

From these may be derived

$$\tan \phi'' = \tan d \sec t$$

$$\cos \phi' = \sin \phi'' \sin h \operatorname{cosec} d.$$

It is evident from the figure that

$$L = \phi'' \sim \phi'$$

There are four cases:

I. Declination has the same name and is greater than the latitude, then the latitude is equal to Phi double prime

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[1] "Bowditch," op. cit., edition of 1926, p. 143.

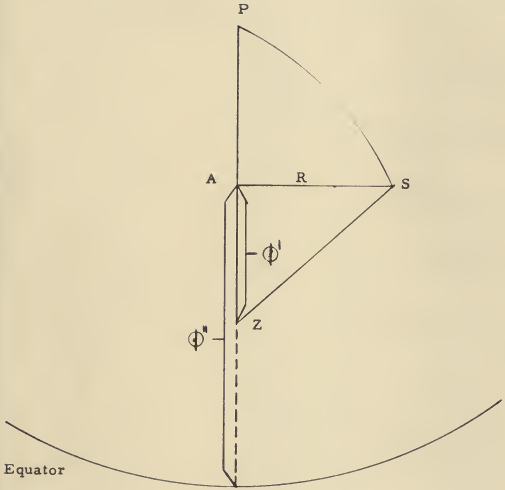

Figure 7

minus Phi prime.

II. Declination has the same name and is less than the latitude, then the latitude is equal to Phi double prime plus Phi prime.

III. Declination has the same name as the latitude and t is greater than six hours, then the latitude is equal to Phi double prime minus Phi prime.

IV. Declination and latitude have contrary names, then the latitude is equal to Phi prime minus Phi double prime.

Captain Thomas H. Sumner (c. 1850) sailed from Charleston, South Carolina, bound for Greenock, Scotland, on November 25, 1837. After passing longitude 21° west he encountered bad weather and was not able to obtain an observation on a celestial body for several days. About midnight, December 17th, his dead reckoning position was about forty miles off Tuskar Light (Latitude 52° 10' North, Longitude 6° 12' West) and soundings indicated that he was approaching land. Because of a wind change, which made the Irish coast a lee shore, he made several tacks to keep his position until daylight. About 10:00 a.m. on the morning of December 18th he observed an altitude of the sun through a break in the clouds.

Using his dead reckoning latitude, which he realized was unreliable, he determined his chronometer longitude, This gave him a position fifteen miles east of his dead reckoning position. He then

assumed a second latitude ten minutes north of his dead reckoning position and toward the coast. This calculation gave him a position twenty-seven miles ENE of the first position. A third latitude was then assumed ten miles further north which gave him a third position twenty-seven miles ENE of the second position. Upon plotting these three positions on his chart he noticed that they were on a straight line which also passed through Small's Light (Latitude 51° 43' North, Longitude 5° 40' West).

Captain Sumner realized that the altitude he had observed must have been the same at all three computed points, and also at Small's Light, and that his ship must be somewhere on the line through the four points. He kept the ship on a course of ENE and in less than an hour Small's Light came into view. Thus he found that his D.R. position was eight miles too far south and forty-five miles too far west with a dangerous coast ahead. [1]

Captain Sumner published his method in 1843, but it did not find immediate favor. The computations were long and tedious, and navigators preferred the Time Sight, Meridian altitude, Polaris combination.

If two observations are made on two different celestial bodies, or if a second observation is made on the same celestial body after

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[1] "Bowditch," op. cit., edition of 1933, p. 159.

sufficient time has passed so that the azimuth has changed approximately thirty degrees, the two lines of position will intersect at a point called a "fix."

If a fix is obtained from observations on the same celestial body it is known as the Double Altitude Method. As Lecky pointed out the solution when worked out by Sumner's method is a formidable problem and "the rules at the finish are so complicated as to scare most ordinary seafaring men."[1]

Lecky deplored the drawing of lines on a chart and advised that "the whole thing be done by calculation from the first to the last."[2]

In lieu of Sumner's method of calculation, he proposed the method of A. C. Johnson, who had been a Naval Instructor aboard the British training ship Britannica. Sumner's method requires about 530 figures, Johnson's about 280, or nearly one-half as many.

That this method is complicated may be seen from Lecky's explanation of the procedure to follow. Using the D.R. latitude determine the longitude of the first observation. For the second observation, correct the first D.R. latitude for the run of the ship and use this latitude to obtain the second longitude. From an azimuth table find the

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[1] S. T. S. Lecky, Wrinkles in Practical Navigation (New York: D. Van Nostrand Co., Inc., 22nd edition, 1937) p. 504.

[2] Ibid., p. 504.

bearing of the body for each observation. With the latitude and bearing enter a special table, called table C (C is the product of the cotangent of the azimuth and the secant of the latitude). The two numbers obtained are labeled *a* and *b*. They are either added or subtracted according to special rules and the result divided into the difference of the longitudes to obtain the correction for the second latitude. The product of this correction and the value of *a* gives the correction to be applied to the first longitude, and the product with the value of *b* gives the correction to be applied to the second longitude, both applied so the two longitudes agree.[1]

In 1853 G. F. Martelli completed tables which were published twenty years later under the title *Tables of Logarithms.*[2] There are five separate tables which enable a navigator to determine his longitude by an observation on a celestial body. The tables are based on the formula

$$\operatorname{cosec}^2(1/2\ t) = \frac{\cos L \cos d}{1/2 \cos (L \sim d) - \sin h}$$

where t, L, d, and h have the usual meaning.

To use the tables the procedure is as follows:

1. Enter Table I with the latitude and take out the number corresponding.

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[1]Lecky, *op. cit.*, p. 507.

[2]G. F. Martelli, *Tables of Logarithms* (New Orleans: Lightening Printing Office, 1873)

2. Enter Table I with the declination and take out the number corresponding.

3. Enter Table II with the sum of the latitude and the declination if contrary names, with the difference if the same name, and take out the number.

4. Enter Table III with the corrected altitude and take out the number.

5. Add the numbers obtained in steps 3 and 4.

6. Enter Table IV with the result obtained in step 5 and add the value thus obtained to those found in steps 1 and 2.

7. With this sum enter Table V and take out the value of t.

Table I corresponds to the logarithms of the cosines of angles from 0° to 90° for every minute to which .5 has been added and the result multiplied by 10^4.

Table II corresponds to a table of natural cosines to which .2 has been added and this result multiplied by 10^3. This is marked "seconds" and tabulated as "minutes" and "seconds" there being 60 "seconds" to each "minute." For example, cos 27° 36' = .8862. $(.8862 + .2000)10^3$ =1086."2 or 18' 06".2. This is the value tabulated in Table II corresponding to 27° 36'.

Table III corresponds to a table of natural sines multiplied by 10^3 and marked "seconds." This value is then subtracted from 16' 40" (i.e. 1000 "seconds") and changed to "minutes" and "seconds." For example 10^3 sin 15° 38' = 10^3 (.2695) = 269.5. The difference between this value and 1000 is 730.5 "seconds" or 12' 10."5, the value

tabulated in Table III corresponding to 15° 38'. This scheme replaced subtraction by addition.

If we let $\cos A = a$ and $\cos B = b$, the combined results of Tables II and III are

$$1000(a + .2000) + (1000 - 1000\,b)$$

or

$$1000(a - b) + 1200.$$

This means that the number which is used to enter Table IV is 1200 "seconds" or 20 "minutes" too large. Since the maximum value in Table II is 20' 00", and in Table III the maximum value 1000" or 16' 40", values in Table IV need only be tabulated to 2200" or 36' 40". However, additional values are given to complete the page.

The values in Table IV, therefore, are obtained by changing the values of "minutes" and "seconds" to "seconds," deducting 1200, dividing this result by 2000, and finding the cologarithm of the number obtained. To this cologarithm, Martelli added .0334. For example, 24' 51" .1 or 1491".1 less 1200 divided by 2000 is .14556. The cologarithm of this number plus .0334 is .8704, the value tabulated by Martelli corresponding to 24' 51".1.

Since Table I is entered twice, once with the latitude and once with the declination, and since .0334 is added to the cologarithms of the denominator, the sum of the three numbers obtained in step 6 is 1.0334 too large. Consequently the values tabulated in Table V are

log cosec^2 $(1/2t) = 1.0334$. Thus for $t = 42^\circ$, log cosec^2 21° is 0.8913. If 1.0334 is added the result is 1.9247, the value found in Table V corresponding to the 42° or $2^h\ 48^m$.

From 1844 to 1861 the United States Observatory and Hydrographic Office was under the direction of Lieut Matthew F. Maury. During this period much research was done not only in astronomy and hydrography, but also in marine meteorology. The results were published in Maury's Wind and Sailing Charts (1850) and Sailing Directions (1851). These were forerunners of the present day pilot charts of the oceans issued by the United States Hydrographic Office and of the various Sailing Directions.

In 1866 the hydrographic and meteorological branches were separated from the Naval Observatory and placed under the supervision of the present Hydrographic Office. In 1906 the meteorological branch was transferred to the Weather Bureau.[1]

Adolphe Laurent Anatole Marq de Blonde de Saint-Hilaire (1833 - 1889), an Admiral of the French Navy, published an article entitled "Calcul du Point Observé, Methode des Hauteurs Estimées" in 1875, in which he described his method of obtaining the altitude difference and the computed point.

At any given instant of time a celestial body is at the zenith of

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[1]Curtis, op. cit., p. 133.

some location on the surface of the earth. This place may be called the sub-celestial point. If two observations are made simultaneously on the same celestial body from two different places, the altitudes will be the same or will differ. If the altitudes are the same, then the observers are on the circumference of a circle whose center is the sub-celestial point and whose great circle radius in nautical miles is the complement of the observed altitude in minutes of arc. If the altitudes are different, which is usually the case, then the observers are on the circumferences of two concentric circles whose center is the sub-celestial point, the one with the greater altitude being the nearer to the sub-celestial point. These circles are called Circles of Position.

Saint-Hilaire's method consists of comparing the observed altitude with the altitude computed from a point near the true position, called the assumed position. The difference in the altitudes is called the Intercept. Since the assumed position will usually be within thirty miles of the true position, the bearings of the body from the two positions are practically the same.

To detemine the line of position graphically, a line is drawn on the chart in the direction of the sub-celestial point through the assumed position. On this line an intercept is layed off according to the desired direction, toward or away from the sub-celestial point, equal in length to the difference of the altitudes. The line of position is then drawn

through this point perpendicular to the azimuth.

Sir William Thomson, later to be Lord Kelvin, a professor of Natural Philosophy in the University of Glasgow, realized the value of the Sumner line and the advantage to be gained if the work of the navigator could be reduced. He pointed out that if the solutions of the hour angle could be tabulated corresponding to all possible values of each of the three sides of the navigational triangle from 0° to 90°, no calculations would be necessary and the desired data obtained by inspection. For accurate navigation each of the three sides would have to be tabulated from every minute of arc. This would require "the solution of 157,464,000,000 triangles, or at the rate of 1000 triangles a day would take 400,000 years."[1]

As a substitute for this overwhelming task, Thomson tabulated the solutions of 8100 right spherical triangles which he published.[2] His is the first known method to divide the navigational triangle into two right triangles by dropping a perpendicular from the celestial body to the meridian of the observer.

Figure 8, page 78, represents the navigational triangle projected on the plane of the celestial horizon. SO is perpendicular to PZ, where

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[1]Sir William Thomson, Tables for Facilitating Sumner's Method at Sea (London: Taylor and Francis, 1876) p. 1.

[2]Ibid., p. 1.

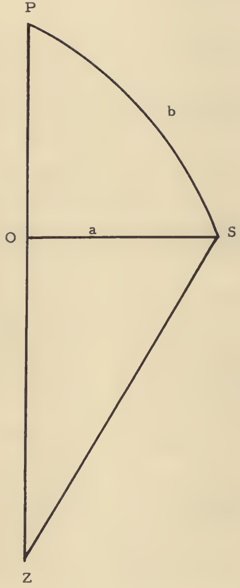

Figure 8

P is the elevated pole, Z is the zenith, and S is the position of the celestial body. Values of a (SO) and b (PO) are tabulated for every degree. Instead of tabulating SP and SZ, Thomson tabulated the complements of these values so as to give the declination and altitude. They were tabulated in columns headed "Co-hyp." The values of the angles opposite SO, that is angle P, the meridian angle, and angle Z, the azimuth, were tabulated in a column headed A.[1]

There were ten rules to follow, some of which were complicated. As an example, Rule VI reads,[2]

> Halve the estimated co-latitude. Taking the tables, look for the number thus found in any of the columns headed "b". If the estimated co-latitude is an odd number of degrees, look for the positions midway between the two numbers above and below the estimated co-latitude. Level with the position so found, and in any one of the columns headed "Co-hyp", place the end of one leg of a pair of compasses, and search from column to column until two numbers are found, both in a column headed "Co-hyp," given by the end of the other leg at equal distances below and above the center position, one of which agrees approximately with the declination and the other with the altitude. The numbers level with these on the right-hand side of the contiguous column headed "A" are approximately the hour angle from the meridian of the ship and the azimuth; that level with the declination being the hour angle, and that level with the altitude being the azimuth. The opening between the legs of the compasses may be varied; it is only necessary that the same distance be taken above and below the level of the estimated half co-latitude.

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[1]Sir William Thomson, op. cit., p. 2.

[2]Ibid., p. 2.

To get the proper opening for the compasses, [1]

Place the end of one leg of a pair of compasses on the middle of the O at the top of any one of the columns headed "b", and open the legs until the other rests on the number in the same column equal to the estimated co-latitude. Lift the compasses, keeping this opening between the legs constant, and place the points on two numbers in one of the columns headed "Co-hyp." Search thus through the "Co-hyp" columns until two numbers are found, one agreeing approximately with the altitude and the other with the declination.

In the same year in which his tables appeared, 1876, Sir William Thomson perfected a new type of dry card compass. It was adopted by the British Navy and used until 1906 when it was supplanted by a liquid type. [2]

The United States Navy had used a liquid type for about ten years prior to Thomson's invention. As it was satisfactory they did not change. In 1813 Francis Crowe had proposed that the compass bowl be filled with a liquid to damp out oscillations of the needle, but not until E. S. Ritchie had perfected a float that took most of the weight off the pivots, in 1862, and W. R. Hammersley invented the expansion chamber, in 1866, were liquid compasses acceptable. [3]

The first azimuth tables to be published by the United States

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[1] Sir William Thomson, op. cit., p. 2.

[2] "Compass," Encyclopedia Britannica, 1952 edition, Volume VI, page 171.

[3] "Compass," Encyclopedia Britannica, edition of 1952. Volume VI, p. 171.

Hydrographic Office appeared in 1881 under the title Artic Azimuth Tables (H. O. #66). These tables give the true bearing of the sun and other celestial bodies within the limits of the ecliptic for integral values of latitude and declination between 70° and 88°, at intervals of ten minutes of time. The tables are entered using latitude, declination, and apparent time as arguments, to find the azimuth.

The following year, 1882, H. O. #71, Azimuths of the Sun, known as the "Red" Azimuth Tables, was published. These are similar to H. O. #66. The true bearing of the sun is given at intertals of ten minutes of time from sunrise to sunset for integral values of latitude from the equator to 71°, and for integral values of declination from 0° to 23°. These tables are divided into three parts; Part I tabulates azimuths for latitude 0° and declinations either north or south; Part II tabulates azimuths for latitudes from 1° to 70° inclusive for declinations of the same name as the latitude; Part III tabulates azimuths for declinations of contrary name.

The next tables of altitudes to follow those of Thomson were by F. Souillagouet, published in France in 1891. These were the first to divide the navigational triangle into two right triangles by dropping a perpendicular from the zenith to find the altitude. Azimuths are computed from tables based on dropping a perpendicular from the celestial body as in Thomson's method, but different formulas are used.[1]

[1]F. Souillagouet, Tables du Point Auxiliare (Toulouse, France: Imprimerie-Douladoure - Privat, 1891)

Several contributions to navigation were made during the nineteenth century, including the publication of a) The New American Practical Navigator by Nathaniel Bowditch, b) Tables of Logarithms by G. F. Martelli, c) Maury's Wind and Sailing Charts and Sailing Directions, d) Artic Azimuth Tables (H. O. #66) and Azimuths of the Sun (H. O. #71).

The splitting of the astronomical triangle into two right triangles first done for navigation purposes by Sir William Thomson in which the perpendicular was dropped from the body to the observer's meridian, and by F. Souillagouet in which the perpendicular was dropped from the zenith to the hour circle of the body, were also notable contributions.

The outstanding contributions, however, were the discovery of the Line of Position by Captain Charles Sumner which bears his name, and the intercept method of locating this line by Admiral Saint-Hilaire.

CHAPTER VI

FROM 1900 TO THE END OF WORLD WAR II

In 1902 the Hydrographic Office published a third set of azimuth tables entitled Azimuths of Celestial Bodies (H. O. #120), the "Blue Azimuth Tables." These tables give values of the azimuths of celestial bodies tabulated at ten minute intervals from $0^h\ 00^m$ to $12^h\ 00^m$ for integral values of declinations from 24^o to 70^o for latitudes and declinations of the same name. They are entered with the same arguments as H. O. #66 and H. O. #71.[1] They may be used if the latitude and declination are of contrary name by entering with the supplement of the hour angle. The supplement of the tabulated azimuth is then the required true bearing.

These azimuth tables may be used to determine a Line of Position. Using an assumed latitude compute the corresponding longitude. Through this point the line perpendicular to the azimuth as found in the appropriate table is a Line of Position.

If values other than integral latitudes and declinations are used, a double interpolation is necessary.

Since the publication of the Thomson and Souillagouet tables many others have appeared. In general they may be classified

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[1]Infra., p. 84.

according to the purpose and method used. Soule and Collins[1] have grouped them as

1. Longitude methods using the D. R. Position.

2. Longitude methods using an assumed position.

3. Altitude methods not dividing the triangle.

4. Altitude methods using two right triangles formed by dropping a perpendicular from the celestial body.

5. Altitude methods using two right triangles formed by dropping a perpendicular from the zenith.

6. Altitude methods using the D. R. position.

7. Altitudes tabulated from an assumed position.

Several of these tables are listed in the Bibliography[2] and in the Chronological Table.[3]

In 1905 the Davis Tables published in England were the first to tabulate in adjacent columns values of natural haversines and their logarithms. The cosine-haversine formula

$$\text{hav } z = \text{hav } (L \sim d) + \cos L \cos d \text{ hav } t$$

where

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[1] C. C. Soule and E. B. Collins, "Resume of Navigation Methods." (Washington: U. S. Hydrographic Office. Supplement to the Pilot Chart of the North Atlantic Ocean, 1934)

[2] Supra, p. 108.

[3] Supra., p. 100.

z = the complement of the computed altitude

L = the latitude of the observer

d = the declination of the body

t = the meridian angle,

now became very popular. The second term in the formula is computed by logarithms. The anti-logarithm of this is found in the haversine table. To this value the natural haversine of $(L \sim d)$ is added and the value of z is determined from the same table.

With the printing of a similar table in the 1914 edition of Bowditch, the cosine-haversine formula was widely used by United States navigators for many years.

In 1920 the Japanese Hydrographic Office published New Altitude and Azimuth Tables, Between Latitudes 65°N and 65°S., for the Determination of the Position Line at Sea, the work of S. Ogura of that office. The tables for altitude were based on the formula

$$\operatorname{cosec} H = \sec N \sec (K \sim d).$$

Figure 9, page 86, shows the navigational triangle projected on the plane of the celestial horizon, where

P = the elevated pole

Z = the zenith of an assumed position

M = the celestial body

N = ZA = the perpendicular from the zenith to the meridian of the body.

K = the distance AB

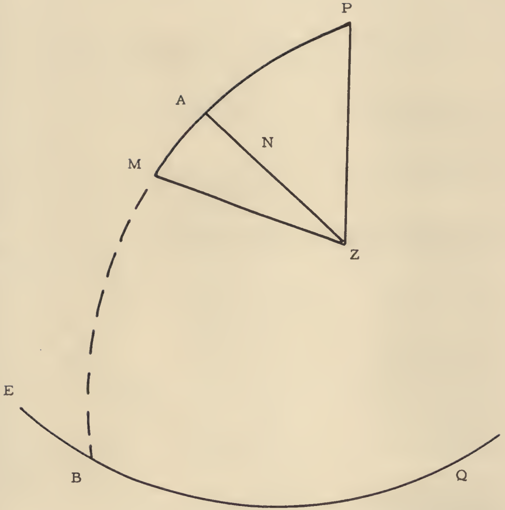

Figure 9

EQ = the equator

Table A tabulates values of log sec N multiplied by 10^5 in a column designated A, and values of K for integral values of latitude from 0° to 65° and for integral values of the hour angle.

Table B is a table of log secants and log cosecants multiplied by 10^5.

These altitude tables were utilized by P. V. H. Weems in his Line of Position Book published in 1927.[1] Incorporated in these tables is Rust's Azimuth diagram which was first published by Captain Armistead Rust of the United States Navy in 1918.[2]

The second edition published in 1928 extended Ogura's Table A to include latitudes to 90°. These were computed especially for Mr. Lincoln Ellsworth and Commander Richard E. Byrd for their contemplated polar flights.

A Hydrographic Office publication that has gone through many editions is Navigation Tables for Mariners and Aviators (H. O. #208) by J. Y. Dreisonstok. The first edition was printed in 1928. Figure 10, page 88, shows the navigational triangle as used in these tables, projected on the plane of the celestial horizon.

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[1] P. V.H. Weems, Line of Position Book (Annapolis, Maryland, Weems School of Navigation, fourth edition, 1943) p. III.

[2] Armistead Rust, Practical Tables for Navigators and Aviators (Philadelphia: John E. Hand, 1918).

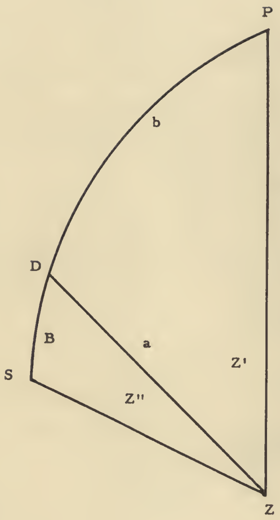

Figure 10

P = the elevated pole

Z = the zenith of an assumed position

ZD = the perpendicular from Z to PS

S = the celestial body.

The distance SD is called B, the distance DP is called b, the length of the perpendicular is called a, and the two angles into which the azimuth is divided are called Z' and Z'' as shown.

In the triangle PZD, from Napier's Rules of Circular Parts,

1. $\sin a = \cos L \sin t$

2. $\tan b = \cot L \cos t$

3. $\cot Z' = \sin L \tan t$

In the triangle DSZ

4. $\sin h = \cos a \cos B$

5. $\cot Z'' = \sin a \cot B$

These are transformed into

6. $\sin h = \cos a \sin (d + b)$

7. $\cot Z'' = \sin a \tan (d + b)$

and inverted to obtain

8. $\operatorname{cosec} h = \sec a \operatorname{cosec} (d + b)$

9. $\tan z'' = \operatorname{cosec} a \cot (d + b)$

Table I and IA give for every degree of latitude from 0° to 90° and for every degree of local hour angle from 1° to 360° four columns labeled b, A, C, and Z'.

Column b is the value of side b in degrees, minutes and tenths found from equation 2.

Column A is the log sec a multiplied by 10^5 found from equation 1.

Column C is the log cosec a to three decimal places multiplied by 10^3 found from equation 3.

Column Z' is the value of that part of the azimuth indicated in the figure found from equation 3.

Table II is a table of log cosecants and log cotangents of angles from 0° to 180° tabulated at intervals of one minute. It contains two columns labeled B and D.

Column B is the log cosecant of angles multiplied by 10^5. Opposite B is a number which is the average difference of the logarithms in that column for one minute.

Column D is the log cotangent of the same angles to three places multiplied by 10^3. At the top of each page is found the average value for B and D for that page.

To use the tables, Table I (or IA) is entered using as arguments the meridian angle obtained by assuming a longitude so that the angle is integral, and a latitude which is the D.R. latitude assumed to the nearest degree, to obtain values of b, A, C, and Z'. The value of b is then combined with the declination of the observed body. With this quantity Table II is entered, and values of B and D found.

With A + B enter column B of Table II. The result is the computed

altitude (equation 8).

With C + D enter column D of Table II. The result is Z" (equation 9).

Z' and Z" are added to obtain the azimuth.

Equally popular is another Hydrographic Office publication, Dead Reckoning Altitude and Azimuth Tables (H. O. #211) by A. A. Ageton, published in 1932.

These tables are actually tables of log secants and log cosecants multiplied by 10^5 tabulated for all angles from 0° to 90° at intervals of thirty seconds. The two columns are designated A and B, A being the values of the log cosecants, B the value of the log secants.

The altitude and azimuth are computed from the D. R. position. No interpolation is necessary. The azimuth is easily determined from the tables, and the solution is short, simple, and uniform under all conditions.

To accomplish this, Ageton dropped a perpendicular from the celestial body to the observer's meridian as shown in Figure 11, page 92.

P = the elevated pole

Z = the observer's zenith

S = the celestial body

R = the length of the perpendicular SX dropped from the body to the observer's meridian.

t = the meridian angle SPZ

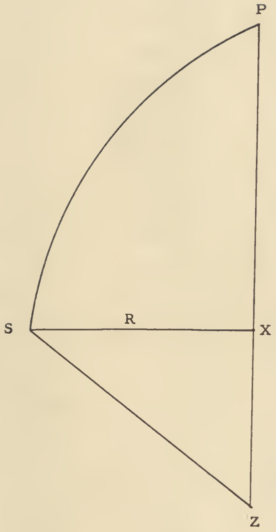

Figure 11

$Z =$ the azimuth

$K =$ the distance of the body from the equator

Using Napier's Rules of Circular Parts and inverting, Ageton obtained the formulas,

1. $\operatorname{cosec} R = \operatorname{cosec} t \sec d$

2. $\operatorname{cosec} K = \operatorname{cosec} d\ (1/\sec R)$

3. $\operatorname{cosec} H = \sec R \sec (K \sim L)$

4. $\operatorname{cosec} Z = \operatorname{cosec} R\ (1/\sec H)$

Supplements to the Nautical Almanac were issued in 1930 and 1931, listing for the first time, the Greenwich hour angle of the moon. In 1932 this feature became a part of the regular almanac, and in 1934 the Greenwich hour angles of the sun, moon, Venus, Mars, Jupiter, Saturn, and fifty-five stars were tabulated.

The first Air Almanac was published by the United States Naval Observatory for the year 1933. This almanac tabulated the Greenwich hour angles and the declinations of the sun for every day of the year at intervals of one hour, for the moon at intervals of ten minutes, of Venus, Mars, Jupiter, and Saturn for 0^h, Greenwich Mean Time, daily, and for fifty stars and the First Point of Aries for the same time. An additional 144 stars were listed and their Greenwich hour angles and declinations were given for 0^h on the first of each month. A special table for Polaris was also included. The A and B values of these 194 stars, corresponding to the log cosecants and the log secants

were given for the convenient use of H. O. #211. Correction tables to facilitate interpolations were included. This was an experimental volume, the next appearing in 1941. Since that time the Air Almanac has been published for each year.

The first volume of the Hydrographic Office publication Tables of Computed Altitude and Azimuth (H. O. #214) was published in 1936. This was volume IV covering latitudes from 30^{o} to 39^{o} inclusive.

These tables consist of the tabulated solutions of the navigational triangle, each volume covering ten degrees of latitude.

The tables are entered with the nearest integral degree of latitude, nearest integral or one-half degree of declination, and nearest integral degree of meridian angle to find values of the altitude to the nearest tenth of a minute and the azimuth to the nearest tenth of a degree. The altitude is corrected for the difference between the tabulated declination and the true declination to the nearest minute. To accomplish this, values of a quantity called "delta d" are tabulated adjacent to each altitude. This value is the cosine of the position angle. No correction need be made to the tabulated azimuth. A value of "delta t" is also given. This enables a navigator to work from his D.R. position by correcting for the difference between the tabulated hour angle and that from the D.R. position. An additional correction for the difference in latitude must also be applied.

During World War II a British Publication was electrotyped and

issued in this country as H. O. #218. This is similar to H. O. #214. It was devised primarily for air navigation. Tabulated values of altitude and azimuth are given to the nearest minute and degree respectively. This publication consists of thirteen volumes, each covering five degrees. The complete range is from S. 64^{o} to N. 64^{o}. The main difference between these two tabulated solutions is that H. O. #218 contains the tabulated altitude and azimuth for twenty-two selected stars, computed for the epoch of 1940. A second part contains the tabulated solutions for all integral values of declinations from 0^{o} to 28^{o} thus enabling the tables to be used for the sun, moon, planets, and other stars within the ecliptic limits. A refraction correction at 5000 feet is included in the tabulated altitudes.

CHAPTER VII

CONCLUSION

Through the centuries navigation advanced very slowly. The Phoenicians developed it as an art and for over two thousand years an art it remained. Not until the fifteenth century, under the leadership of Prince Henry, of Portugal, did it become a science.

Improvements in instruments and methods were slow in developing, and in many instances several years were to pass before these improvements or new devices were accepted.

Mercator's chart was found wanting until accurate positions were obtained. The heliocentric theory of Copernicus had to wait nearly one hundred years before it displaced the geo-centric theory of Ptolemy. However, in the field of navigation, the Ptolmaic postulate of a stationary world with the celestial system revolving about it is still utilized.

The theory that longitude could be determined from the variation of the compass was held for over one hundred years before it was finally discarded as false.

The method of lunars, suggested by John Werner in 1514, promoted by Maskelyne two hundred fifty years later, continued to be practiced until the early part of the twentieth century. Chronometers were available and reliable, but were very expensive. Lunars did not

become obsolete until a time check from shore could be obtained by wireless telegraph, about 1910. The 1912 edition of Bowditch was the first to omit the lunar method, but for several years it had appeared as Appendix IV.

Saint-Hilaire's intercept method, first published in 1875, was slow to be adopted. At the United States Naval Academy the first mention in text books used there was in 1906,[1] but instruction in the older methods continued until after 1930.[2]

As late as 1944 the older methods persisted. The Secant Time Sight Tables by Weems were published in that year. During World War II Commander Weems became convinced that nearly one-quarter of the Merchant Marine navigators would continue to use the time sight method due to "tradition, aversion to plotting, influence of examiners, and to other causes."[3]

In 1947 the United States Hydrographic Office published a new set of tables under the title Star Tables for Air Navigation, H. O. #249. These tables were conceived and designed by Commander C. H. Hutchins of the United States Navy, while on duty at the Navigation School

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[1]W. A. Mason, "Marq Saint-Hilaire, Father of the New Navigation," U. S. Naval Institute Proceedings, Volume LXV, 1939, p. 1175.

[2]Ibid., p. 1176.

[3]P. V. H. Weems, The Secant Time Sight, (Annapolis, Maryland: Weems' System of Navigation, 1944) p. 14.

at Pensacola in 1941. They are described in the Proceedings of the United States Naval Institute in the August 1942 issue.[1] One volume, about the size of H. O. #214 contains all integral values of latitude from pole to pole. Six selected stars are listed horizontally, arranged in order of ascending azimuth, with their computed altitudes and true azimuths for each integral value of the local hour angle of the First Point of Aries, except from latitude 70° to the poles, in which case the tabulation is for 2° intervals. The stars were selected on a basis of magnitude, altitude and azimuth. All of the first magnitude stars except Beta Crucis are used, together with nineteen additional stars of second magnitude. For the complete tables a total of thirty-eight stars were used. The tabulated altitudes contain a correction for refraction at 5000 feet. There are seven supplementary tables.

The star tables are entered with an integral value of local hour angle of Aries (an even value if the latitude is greater than 69°) and a latitude of integral value nearest the D. R. position, to obtain the computed altitude and azimuth.

This was followed in 1951 by another edition in three volumes under the title Sight Reduction Tables for Air Navigation. Volume I is similar to the preliminary edition, but is computed for the epoch of 1955. Volumes II and III cover latitudes from 0° to 39° and from 40° to 89° respectively, for declinations from 0° to 29°, and are therefore

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[1] C. H. Hutchins, USNIP, Volume LXIII, p. 1279.

designed to be used for the sun, moon, planets, and for stars that lie within the limits of the ecliptic. These three volumes will replace the thirteen volumes of H. O. #218. Surface vessels, for the most part, will probably continue to favor H. O. #214. During the last war, submarine navigators found H. O. #218 very useful. H. O. #249 will be found of equal value.

Other aids and methods devised especially for air navigation have proved to be useful aboard ships. The American Air Almanac, used by practically all American air navigators, has been adopted by many marine navigators.

The magnetic compass and the marine gyro compass lose their effectiveness as the poles are approached. An electric directional gyro compass has been developed to be used in these regions. Special charts have been devised for polar navigation, but are inadequate at the present time because accurate surveys have yet to be completed.

The history of navigation shows that it is an art based upon mathematics, astronomy, geography, physics, and the development of scientific instruments. It has taken many centuries to accomplish the high state of perfection which has now been reached. In pushing forward the frontiers of navigation, man has learned much about the world on which he lives.

CHRONOLOGICAL TABLE

Most of the material in the following chronological table has been selected from similar tables in Smith's History of Mathematics[1] and Chambers' A Handbook of Descriptive Astronomy.[2] The more recent events have been obtained from the original sources.

Date B.C.	Event
c. 3800	Maps were used in Babylon.
c. 3100	Egyptians voyaged on the Mediterranean.
c. 2300	Cadastral surveys made in Egypt.
c. 1500	Phoenicians engaged in trade.
c. 600	Thales of Miletus suggested spherical earth.
	Periplus of Sylax probably compiled.
	Phoenicians circumnavigated Africa.
c. 550	Anaximander made first known map of the world.
	Pythagoras taught the earth is a sphere.
517	Hicataeus wrote the first geography.
500	Parmenides taught the earth is a sphere.
380	Plato taught geometry in Athens.

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[1] Smith, D. C., op. cit., Volume I, pp. 549 - 570.

[2] Chambers, op. cit., pp. 762 - 769.

Date B.C.	Event
330	Pytheas noticed relation between the tides and the sun and moon.
300	Euclid assembled his books of geometry.
250	Dicaearcus drew the first parallel across a map.
230	Eratosthenes measured the circumference of the earth.
225	Aristarchus suggested a spherical earth rotating on its axis.
150	Hipparchus discovered precession; used right ascension and declination and later latitude and longitude; formed first regular catalogue of stars.
	Crates of Mallus divided a globe into four inhabitable quarters.
100	Posidonius measured the circumference of the earth.
25	Strabo wrote the first general treatise on geography.
A.D.	
150	Ptolemy announced his geo-centric theory.
710	The Venerable Bede recognized the Establishment of the Port.
880	Albatani first used sines and chords.
980	Abul-Wefa first used secants, tangents, and cotangents.
1080	Geber introduced cosines.
1150	Alexander Neckam described a magnetic needle.
1250	Approximate date of Sacrobosco's Sphaera Mundi.
1252	Isaac ben Sid edited the Alfonsine Tables.

Date A. D.	Event
1269	Petrus Perigrinus de Maricourt described a compass.
1380	Da Buti described a compass of the form used in modern times.
1400	Prince Henry of Portugal became interested in navigation.
1436	First traverse table based upon trigonometric functions appeared.
1450	Printing from movable type invented.
1455	First almanac printed.
1472	The Sphaera Mundi printed.
1474	Regiomontanus corrected the obliquity of the ecliptic to 23° 30'.
1481	King John II of Portugal appointed his Mathematical Junta.
	Abraham Zacuto arrived in Portugal with his Almanac Perpetuum.
1492	Columbus discovered America.
1509	Regiomento do Estrolabio y do Quadrante published.
1514	John Werner suggested the method of lunars to determine longitude.
1530	R. Gemma Frisius suggested a "Lyttle clocke" to determine longitude.
1537	Pedro Nunes invented the "nonius."
1543	Copernicus announced his heliocentric theory.
1551	Martin Cortes published text on navigation.
	Erasmus Reinhold published first tables based on the theory of Copernicus.

Date A.D.	Event
1569	Gerhard Kramer exhibited his Mercator projection.
1578	Humfray Cole invented the patent log.
1581	Michael Coignet described nocturnals.
1590	Simon Stevin invented decimal fractions.
1595	Captain John Davis published the Seamans Secrets.
1598	King Philip III of Spain offered a reward for the solution of the problem of the determination of longitude.
1599	Edward Wright developed the mathematical theory of the Mercator projection.
1609	Kepler announced his first two laws.
1610	Galileo discovered four of Jupiter's satellites.
1614	Napier invented logarithms.
1615	Henry Briggs used ten for a base of logarithms.
1618	Kepler announced his third law.
1620	Edmund Gunter published the first tables of logarithms of sines and tangents.
1625	Addison published Arithmetia Navigation using logarithms.
1627	Kepler published the Rudolphine Tables.
1631	Pierre Vernier invented his device.
	Johann Hevelius invented the tangent screw.
1633	Galileo forced to deny the Copernician theory.
1635	Henry Gellibrand discovered annual variation.

Date A.D.	Event
1637	Richard Norwood determined the length of a degree of latitude within an error of about six-tenths of one per cent.
1659	Christian Huygens designed a marine clock.
1665	Major Holmes reported success with pendulum watches.
1675	Greenwich observatory built.
1683	Flamsteed compiled tide tables for London Bridge.
1687	Edmund Halley published Newton's <u>Principia</u>.
1701	Edmund Halley published first variation chart.
1714	Board of Longitude appointed in England.
1730	Thomas Godfrey and John Hadley independently invented the sextant.
1735	John Harrison invented a chronometer.
1761	Harrison perfected chronometer number four.
1763	British Mariner's Guide published.
	Date of the Time Sight.
1767	British Nautical Almanac first published.
	Neville Maskelyne considered the method of lunars the best method for the determination of longitude.
1795	British Hydrographic Office established.
1800	Captain Huddart invented the station pointer.
1802	First edition of the <u>American Practical Navigator</u>.
1803	Captain Matthew Flinders demonstrated the deviation of the compass.

Date A.D.	Event
1827	Lynn's Horary and Azimuth tables published.
1832	First Official British Admiralty Tables.
1837	Captain Sumner discovered his Line of Position.
1843	Sumner published his method.
1850	Maury's Wind and Sailing Charts issued.
1851	Maury's Sailing Directions issued.
1853	Martelli completed his tables.
1857	Chauvenet's approximate lunar method printed in the American Ephemeris and Nautical Almanac.
1862	E. S. Ritchie perfects compass float.
1866	Hydrographic and meteorological brances of the Naval Observatory placed under supervision of the Hydrographic Office.
1875	Saint-Hilaire published his method.
1876	Sir William Thomson published his tables, the first to divide the navigational triangle into two right triangles with a perpendicular from the celestial body.
	Sir William Thomson invented the dry compass.
1881	H. O. #66 published.
	First edition of Lecky's Wrinkles in Navigation published.
1882	H. O. #71 published.
1891	Souillagouet published his tables, the first to drop a perpendicular from the zenith.

Date A.D.	Event
1901	Fuss (German) Tables published by the Russian Hydrographic Office.
1902	Davis' Tables (English) published.
	H. O. #120, Blue Azimuth Tables published.
1905	Davis' cosine-haversine tables published.
1907	Ball's Tables (English) published.
1909	Aquino's Tables published.
	H. O. #127, Star Identification Tables published.
1912	The method of lunars omitted from H. O. #9.
1914	Blackburne's Tables published.
1917	H. O. #200 published.
1918	Captain Armistead Rust's Tables published. Azimuth found from an original diagram.
1919	H. O. #201 published.
1920	H. O. #202, Noon Interval Tables, published.
	Ogura's tables published.
1923	H. O. #203 published.
1925	H. O. #204 published.
1927	Weems' Line of Position Tables published.
1928	H. O. #208 (Dreisonstok) published.
	Weems' Star Altitude Curves published.

Date A.D.	Event
1930	H. O. #209 (Pierce) published.
	First tabulation of the Greenwich Hour Angle in the Nautical Almanac.
1931	Gingrich's tables published.
1932	H. O. #211 (Ageton) published.
1933	First American Air Almanac published.
1936	First volume of H. O. #214 published.
1940	British publication A. P. 1618 issued as H. O. #218.
1943	Weems' New Line of Position Tables published.
1944	Weems' The Secant Time Sight published.
1947	H. O. #249 (Hutchins) Preliminary edition of Star Tables for Air Navigation issued.
1951	H. O. #249 revised, issued in three volumes.

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127	Star Identification Tables (1909).
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BIOGRAPHICAL SKETCH

Walter Priest Morse was born in Berlin, Massachusetts, on April 2, 1902. He graduated from Newton, Massachusetts, High School in 1922 and from the University of Maine in 1926 with the degree of Bachelor of Arts in Mathematics and Astronomy. He received the degree of Master of Arts in Mathematics and Astronomy from the University of Maine in 1928. During the time he was at Maine he was Student Instructor (1925 - 6) and Instructor (1926 - 8) in the Department of Mathematics. In 1928 he became Instructor in Mathematics at Ricker Junior College, Houlton, Maine, and in 1933 was appointed Dean when that office was created. He attended summer sessions at the University of Maine in 1928 and 1931, and Teachers College, Columbia University, in 1935. He served with the United States Navy from 1942 to 1945. In 1948 when Ricker Junior College expanded into a four year liberal arts college he became the Dean of Ricker College. Since February, 1951, he has been pursuing graduate studies at the University of Florida during which time he has been Graduate Assistant (1951 - 2) and Instructor (1952 to present) in the Department of Mathematics. He is a member of the National Council of Teachers of Mathematics.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of the committee. It was submitted to the Dean of the College of Arts and Sciences and to the Graduate Council and was approved as partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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